

January 2005  
6664 Core Mathematics C2  
Mark Scheme

Question Number	Scheme	Marks
1.	$(3+2x)^5 = (3^5) + \binom{5}{1}3^4 \cdot (2x) + \binom{5}{2}3^3(2x)^2 + \dots$ $= \underline{\underline{243+810x+1080x^2}}$	M1 B1, A1, A1 (4) <b>(4 marks)</b>
2.	(a) $\left(\frac{5+13}{2}, \frac{-1+11}{2}\right), = \underline{\underline{(9,5)}}$  (b) $r^2 = (9-5)^2 + (5-(-1))^2 (=52)$ or $r^2 = (13-9)^2 + (11-5)^2 (=52)$ (or equiv.) Equation of circle: $(x-9)^2 + (y-5)^2 = 52$ (or equiv.)	M1, A1 (2)  M1 M1 A1ft A1 (4) <b>(6 marks)</b>
3.	(a) $\log 3^x = \log 5$ $x = \frac{\log 5}{\log 3}$ or $x \log 3 = \log 5$ $= \underline{\underline{1.46}}$  (a) $\log_2\left(\frac{2x+1}{x}\right) = 2$ $\frac{2x+1}{x} = 2^2$ or 4 $2x+1 = 4x$ $x = \frac{1}{2}$ or 0.5	M1 A1 A1 cao (3)  M1 M1 M1 A1 (4) (7)
4.	(a) $5(1-\sin^2 x) = 3(1+\sin x)$ $5-5\sin^2 x = 3+3\sin x$ $0 = \underline{\underline{5\sin^2 x + 3\sin x - 2}}$ *  (b) $0 = (5\sin x - 2)(\sin x + 1)$ $\sin x = \frac{2}{5}, -1$ (both) $\sin x = \frac{2}{5} \Rightarrow x = \underline{\underline{23.6}}$ ( $\alpha = 23.6$ or $156.4$ ) $\phantom{\sin x = \frac{2}{5} \Rightarrow x = \underline{\underline{23.6}}}, \underline{\underline{156.4}}$ ( $180 - \alpha$ )  $\sin x = -1 \Rightarrow x = \underline{\underline{270}}$	M1  A1 cso (2) M1 A1 cso B1 M1 B1 (5) (7) (ignore extra solutions <u>outside</u> the range)

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5.	<p>(a) <math>f(2) = 1 \Rightarrow 8 - 2 \times 4 + 2a + b = 1</math>  <math>f(-1) = 28 \Rightarrow -1 - 2 - a + b = 28</math>  solving <math>\begin{cases} 2a + b = 1 \\ -a + b = 31 \end{cases} \Rightarrow \underline{a = -10, b = 21}</math></p> <p>(b) <math>f(3) = 27 - 18 + 3a + b</math>  <math>= 27 - 18 - 30 + 21 = 0</math> <span style="float: right;"><math>\therefore (x - 3)</math> is a factor</span></p>	M1 A1 M1 A1  M1 A1 (6)  M1 A1 c.s.o (2)  (8)
6.	<p>(a) <math>ar = 7.2, ar^3 = 5.832 \Rightarrow r^2 = \frac{5.832}{7.2} (= 0.81)</math>  <math>r = 0.9</math></p> <p>(b) <math>a = \frac{7.2}{(a)}, = \underline{8}</math></p> <p>(c) <math>s_{50} = \frac{8(1 - (0.9)^{50})}{1 - 0.9}</math>  <math>= \underline{79.588} \text{ (3dp)}</math></p> <p>(d) <math>s_{\infty} = \frac{8}{1 - 0.9} (= 80)</math>  <math>s_{\infty} - s_{50} = 80 - (c) = 0.412</math></p>	M1 A1 (2)  M1, A1 (2)  M1 A1 c.a.o (2)  M1 A1 <sup>^</sup> (2) (8)
7.	<p>(a) <math>r\theta = 8 \times 0.7, = 5.6(\text{cm})</math></p> <p>(b) <math>BC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos 0.7</math>  <math>\Rightarrow BC = 7.098</math>  <math>\Rightarrow \text{Perimeter} = (a) + (11 - 8) + BC, = 15.7(\text{cm})</math></p> <p>(c) <math>\Delta = \frac{1}{2}ab \sin c = \frac{1}{2} \times 11 \times 8 \times \sin 0.7, = \text{AWRT } 28.3</math>  Sector <math>= \frac{1}{2}r^2\theta =, \frac{1}{2} \times 8^2 \times 0.7</math>  Area of <math>R = 28.345 \dots - 22.4 = 5.9455 = 5.95(\text{cm}^2)</math></p>	M1, A1 (2)  M1 A1 M1, A1cao (4)  M1, A1  M1, A1  A1 (5)  (11)

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8.	<p>(a) <math>x^2 + 6x + 10 = 3x + 20</math>  <math>\Rightarrow x^2 + 3x - 10 = 0</math>  <math>(x + 5)(x - 2) = 0</math> so <math>x = -5</math> or <math>2</math>  sub a value for <math>x</math> to obtain a value for <math>y</math> in <math>y = 3x + 20</math>, <math>y = 5</math> or <math>26</math></p> <p>(b) line - curve =, <math>10 - 3x - x^2</math>  <math>\int (10 - 3x - x^2) dx = 10x - \frac{3}{2}x^2 - \frac{x^3}{3}</math>  <math>\left[ 10x - \frac{3}{2}x^2 - \frac{x^3}{3} \right]_{-5}^2 = \left( 20 - \frac{3}{2} \times 4 - \frac{8}{3} \right) - \left( -50 - \frac{3}{2} \times 25 + \frac{125}{3} \right)</math>  <math>= 11\frac{1}{3} - -45\frac{5}{6} = \underline{\underline{57\frac{1}{6}}}</math></p>	<p>M1  M1, A1  M1, A1  (5)</p> <p>M1, A1  M1 A2/1/0<math>\checkmark</math></p> <p>M1  A1  (7)</p> <p>(12)</p>
ALT (b)	<p><math>\int (x^2 + 6x + 10) dx = \frac{x^3}{3} + 3x^2 + 10x</math> (-1 each incorrect term)</p> <p>use of limits = <math>\left( \frac{8}{3} + 12 + 20 \right) - \left( -\frac{125}{3} + 75 - 50 \right) = 51\frac{1}{3}</math></p> <p>Area of Trapezium = <math>\frac{1}{2} (5 + 26)(2 - -5) = 108\frac{1}{2}</math> or <math>46 - - 62.5</math> (from integration)</p> <p>Shaded area = Trapezium - <math>\int = 108\frac{1}{2} - 51\frac{1}{3} = 57\frac{1}{6}</math> or <math>\frac{343}{6}</math> or <math>57.1\dot{6}</math></p>	<p>M1 A2/1/0</p> <p>M1</p> <p>B1<math>\checkmark</math></p> <p>M1 A1  (7)</p>

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9.	<p>(a) Perimeter <math>\Rightarrow 2x + 2y + \pi x = 80</math></p> <p>Area <math>\rightarrow A = 2xy + \frac{1}{2}\pi x^2</math></p> <p><math>y = \frac{80 - 2x - \pi x}{2}</math> and sub in to A</p> <p><math>\Rightarrow A = 80x - 2x^2 - \pi x^2 + \frac{1}{2}\pi x^2</math></p> <p>i.e. <math>A = 80x - (2 + \frac{\pi}{2})x^2</math> *</p> <p>(b) <math>\frac{dA}{dx} = 80 - 2(2 + \frac{\pi}{2})x</math> or <math>80 - 4x - \pi x</math> (or equiv.)</p> <p><math>\frac{dA}{dx} = 0 \Rightarrow 40 = (2 + \frac{\pi}{2})x</math> so <math>x =, \frac{40}{2 + \frac{\pi}{2}}</math> or <math>\frac{80}{4 + \pi}</math> or Awrt 11.2</p> <p>(c) <math>\frac{d^2A}{dx^2} = -4 - \pi</math></p> <p><math>&lt; 0 \therefore A</math> is Max</p> <p>(d) Max Area = <math>80(b) - (2 + \frac{\pi}{2})(b)^2</math></p> <p><math>= \underline{\underline{448(m^2)}}</math> (448 only for A1)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1 c.s.o (4)</p> <p>M1, A1</p> <p>M1, A1 (4)</p> <p>M1</p> <p>A1 (2)</p> <p>M1</p> <p>A1 cao (2)</p> <p>(12)</p>