MATHEMATICS   **9709/13**

Paper 1 Pure Mathematics 1 **(P1)**   **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
                         Graph Paper
                         List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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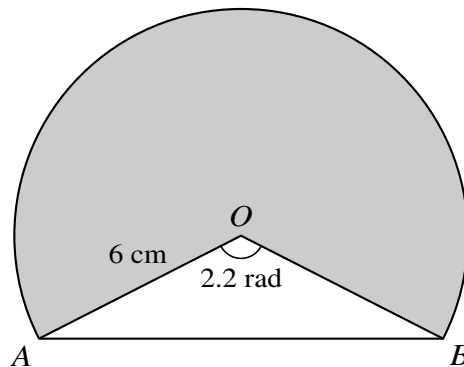
[Turn over

1 Find the coefficient of x in the expansion of $\left(x^2 - \dfrac{2}{x}\right)^5$. [3]

2 The first term in a progression is 36 and the second term is 32.

    (i) Given that the progression is geometric, find the sum to infinity. [2]

    (ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0. [3]

3

    The diagram shows part of a circle with centre O and radius $6\,\text{cm}$. The chord AB is such that angle $AOB = 2.2$ radians. Calculate

    (i) the perimeter of the shaded region, [3]

    (ii) the ratio of the area of the shaded region to the area of the triangle AOB, giving your answer in the form $k : 1$. [3]

4 **(i)** Prove the identity $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$. [3]

    (ii) Hence solve the equation $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} = 3 \sin x - 2 \cos x$ for $0 \leqslant x \leqslant 2\pi$. [3]

5 A function f is such that $f(x) = \dfrac{15}{2x + 3}$ for $0 \leqslant x \leqslant 6$.

    (i) Find an expression for $f'(x)$ and use your result to explain why f has an inverse. [3]

    (ii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1}. [4]

6 A curve is such that $\dfrac{dy}{dx} = \dfrac{12}{\sqrt{(4x + a)}}$, where a is a constant. The point $P(2, 14)$ lies on the curve and the normal to the curve at P is $3y + x = 5$.

    (i) Show that $a = 8$. [3]

    (ii) Find the equation of the curve. [4]

7 The position vectors of points A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}.$$

   (i) Show that angle $BAC = \cos^{-1}(\frac{1}{3})$. [5]

   (ii) Use the result in part **(i)** to find the exact value of the area of triangle ABC. [3]

8 **(i)** Express $2x^2 - 10x + 8$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

   (ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]
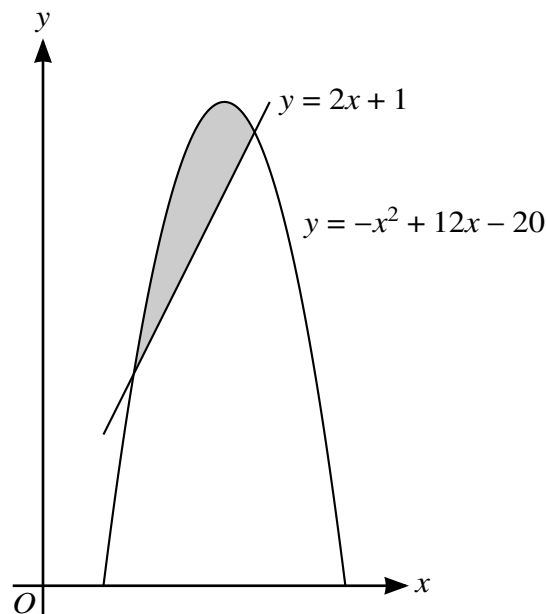
9 The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3.

   (i) Show that the total surface area of the cuboid, A cm^2, is given by

$$A = 6x^2 + \frac{768}{x}.$$ [3]

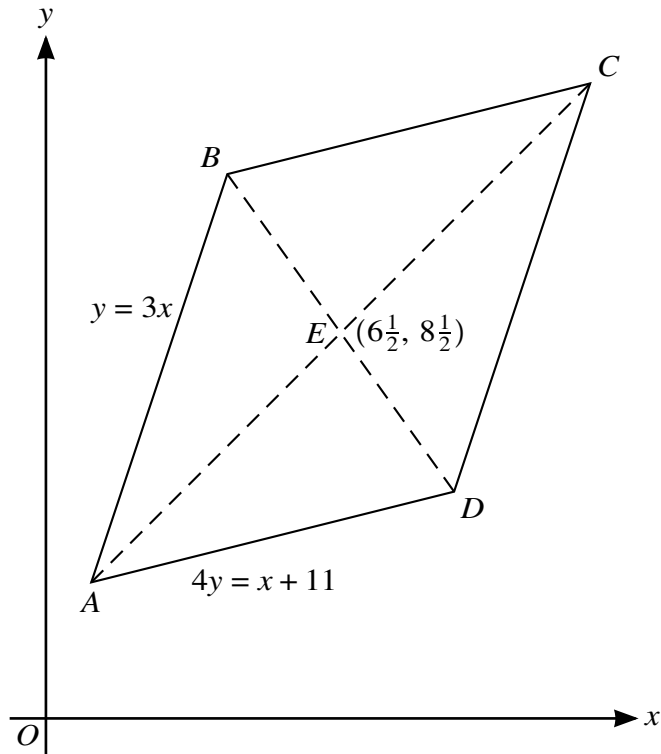   **(ii)** Given that x can vary, find the stationary value of A and determine its nature. [5]

10

The diagram shows the curve $y = -x^2 + 12x - 20$ and the line $y = 2x + 1$. Find, showing all necessary working, the area of the shaded region. [8]

[Question 11 is printed on the next page.]

11

The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y = 3x$ and the equation of AD is $4y = x + 11$. The diagonals AC and BD meet at the point $E\left(6\frac{1}{2}, 8\frac{1}{2}\right)$. Find, by calculation, the coordinates of A, B, C and D. [9]