UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Advanced Subsidiary Level

## MATHEMATICS

9709/22
Paper 2 Pure Mathematics 2 (P2)
May/June 2013
1 hour 15 minutes

Additional Materials: | Answer Booklet/Paper |
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| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 A curve is such that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{4}{7-2 x}$. The point $(3,2)$ lies on the curve. Find the equation of the curve.

2 Solve the inequality $|x-8|>|2 x-4|$.

3 (i) The polynomial $2 x^{3}+a x^{2}-a x-12$, where $a$ is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(x+1)$ is a factor of $\mathrm{p}(x)$. Find the value of $a$.
(ii) When $a$ has this value, find the remainder when $\mathrm{p}(x)$ is divided by $(x+3)$.

4 The variables $x$ and $y$ satisfy the equation $5^{y+1}=2^{3 x}$.
(i) By taking logarithms, show that the graph of $y$ against $x$ is a straight line.
(ii) Find the exact value of the gradient of this line and state the coordinates of the point at which the line cuts the $y$-axis.

5 The equation of a curve is

$$
x^{2}-2 x^{2} y+3 y=9
$$

(i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{2 x-4 x y}{2 x^{2}-3}$.
(ii) Find the equation of the normal to the curve at the point where $x=2$, giving your answer in the form $a x+b y+c=0$.

6 (i) By sketching a suitable pair of graphs, show that the equation

$$
3 \mathrm{e}^{x}=8-2 x
$$

has only one root.
(ii) Verify by calculation that this root lies between $x=0.7$ and $x=0.8$.
(iii) Show that this root also satisfies the equation

$$
\begin{equation*}
x=\ln \left(\frac{8-2 x}{3}\right) . \tag{1}
\end{equation*}
$$

(iv) Use the iterative formula $x_{n+1}=\ln \left(\frac{8-2 x_{n}}{3}\right)$ to determine this root correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

7 (a) Find the exact area of the region bounded by the curve $y=1+\mathrm{e}^{2 x-1}$, the $x$-axis and the lines $x=\frac{1}{2}$ and $x=2$.
(b)


The diagram shows the curve $y=\frac{\mathrm{e}^{2 x}}{\sin 2 x}$ for $0<x<\frac{1}{2} \pi$, and its minimum point $M$. Find the exact $x$-coordinate of $M$.

8 (i) Prove the identity

$$
\begin{equation*}
\frac{1}{\sin \left(x-60^{\circ}\right)+\cos \left(x-30^{\circ}\right)} \equiv \operatorname{cosec} x . \tag{3}
\end{equation*}
$$

(ii) Hence solve the equation

$$
\begin{equation*}
\frac{2}{\sin \left(x-60^{\circ}\right)+\cos \left(x-30^{\circ}\right)}=3 \cot ^{2} x-2, \tag{6}
\end{equation*}
$$

for $0^{\circ}<x<360^{\circ}$.

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