UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced/Advanced Subsidiary Level

MARK SCHEME for the May/June 2006 question paper

9709 MATHEMATICS

9709/01   **Paper 1**

Maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

The minimum marks in these components needed for various grades were previously published with these mark schemes, but are now instead included in the Report on the Examination for this session.

*   CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2006 question papers for most IGCSE and GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
                B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only - often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1   A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1   This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1. $\dfrac{dy}{dx} = -kx^{-2}$ Puts $x = 2$, $m = -3$ $\rightarrow k = 12$	B1 M1 A1 [3]	Negative power ok. Subs $x = 2$ into his dy/dx co.	
2. $\tan 2x = -3$ $2x = 180 - 71.6$ or $2x = 360 - 71.6$ $\rightarrow x = 54.2°$ or $144.2°$	M1 DM1 A1 A1√ [4]	Use of $\tan = \sin/\cos$ with "$2x$" "$2x$" in second quadrant. co. For $90 + 1^{st}$ answer.	
3. (i) $r = 1.05$ with GP 2011 is 11 years. Uses ar^{n-1} $\rightarrow$ \$8 144 (or 8140) (ii) Use of S_n formula $\rightarrow$ \$ 71 034	B1 M1 A1 [3] M1 A1 [2]	Anywhere in the question. This could be marked as $2 + 3$. Allow if correct formula with $n = 10$ co. (allow 3 sf) Allow if used correctly with 10 or 11. co (or 71 000)	
4. $(2+ax)^n$ 1st term $= 2^n = 32$ $\rightarrow n = 5$ 2nd term $= n.2^{n-1}(ax) = -40x$ 3rd term $= n(n-1).\frac{1}{2}.2^{n-2}.(ax)^2$ $\rightarrow a = -\frac{1}{2}$ $\rightarrow b = 20$	B1 M1 M1 A1 A1 [5]	co Allow for both binomial coefficients Allow for one power of 2 and ax co co	
5. $y^2 = 12x$ and $3y = 4x + 6$ Complete elimination of 1 variable. $\rightarrow y^2 - 9y + 18 = 0$ or $4x^2 - 15x + 9 = 0$ solution $\rightarrow$ ($\frac{3}{4}$, 3) and (3, 6) Distance $= \sqrt{(3^2 + 2.25^2)} = 3.75$	M1 A1 DM1 A1 M1A1 [6]	x or y must be removed completely. Must be a 3 term quad – not nec $= 0$. Correct method of solution. co. Correct method including √. co.	

6	(i)	$BX = 6\cos30 = 3\sqrt{3}$ $CX = 6\sin30 = 3$	B1 B1	co co
		$\text{Tan } CAB = \text{opp/adj} = \dfrac{3}{4+3\sqrt{3}}$	M1	Must be tan in correct 90° triangle
		$CAB = \tan^{-1}\left(\dfrac{3}{4+3\sqrt{3}}\right)$	A1 [4]	Answer given – beware fortuitous answers.
	(ii)	Pythagoras with his AX and CX or cosine rule used correctly	M1	For any correct method.
		$\rightarrow AC = \sqrt{52+24\sqrt{3}}$	A1 [2]	Answer given – beware fortuitous answers.

7.	(i)	$\tan(\tfrac{1}{2}x) = 15\div8 = 1.875$ $\rightarrow \tfrac{1}{2}x = 1.081$ $\rightarrow x = 2.16$	M1 A1 A1 [3]	Uses correct 90° triangle and sine. Realises the need to ÷ 2 co
	(ii)	$P = 15 + 15 + r\theta = 30 + 17.3$ $\rightarrow 47.3$	M1 A1 [2]	For $r\theta$ only – θ must be in radians. co
	(iii)	Sector area $= \tfrac{1}{2}r^2\theta = 69.1$ Area of $AOBT = 2\times\tfrac{1}{2}\times8\times15 = 120$ Shaded area $= 120 - 69.1$ $\rightarrow 50.8$ or 50.9	M1 M1 A1 [3]	For use of $\tfrac{1}{2}r^2\theta$. For use of 2 triangles or equivalent. co.

8	(i)	Vector $OD = 4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$	B2,1	One off for each error. Column vectors ok.
		Magnitude $= \sqrt{(4^2+4^2+5^2)} = \sqrt{57}$ $\rightarrow$ Magnitude $= 7.55$m	M1 A1 [4]	Correct use of Pythagoras. Accept $\sqrt{57}$.
	(ii)	Vector $OB = 14\mathbf{i} + 8\mathbf{j}$ $OD.OB = 4\times14 + 4\times8 = 88$ $OD.OB = \sqrt{57}.\sqrt{260}\cos\theta$ $\rightarrow$ Angle $DOB = 43.7°$	B1 M1 M1 A1 [4]	co Use of $x_1x_2 + y_1y_2 + z_1z_2$ for his vectors Used correctly co

9	(i)	$\dfrac{dy}{dx} = \dfrac{4}{\sqrt{6-2x}}$		
		If $x = 1$, $m=2$ and perp $m = -\tfrac{1}{2}$ $\rightarrow y - 8 = -\tfrac{1}{2}(x-1)$ $(2y+x=17)$ $\rightarrow (0, 8\tfrac{1}{2})$ and $(17, 0)$ $\rightarrow M(8\tfrac{1}{2}, 4\tfrac{1}{4})$	M1 A1 DM1 A1 B1√ [5]	Use of $m_1m_2=-1$. A1 co for $-\tfrac{1}{2}$ Any correct form of perpendicular. co. For his answers.
	(ii)	$y = \dfrac{4(6-2x)^{\frac{1}{2}}}{\tfrac{1}{2}\times -2} + c$	B1 M1	For 4, $(6-2x)^{\frac{1}{2}}$ and $\div\tfrac{1}{2}$ and no other f(x) For $\div -2$ (only if no other f(x))
		$\rightarrow$ subs $(1,8) \rightarrow c = 16$	M1A1 [4]	Substituting into any integrated expression to find c.

10. $y = x^3 - 3x^2 - 9x + k$

(i) $dy/dx = 3x^2 - 6x - 9$ M1 A1   Attempt to differentiate. All correct.
       $= 0$ when $x = 3$ or $x = -1$ DM1   Sets a differential to 0.
       $\rightarrow x = 3$, $y = 0$ $\rightarrow k = 27$ A1   co.
                               [4]

(ii) $x = -1 \rightarrow y = 32$ B1√ [1]   For his second value.

(iii) $-1 < x < 3$.   B1√ [1]   Realises the need to look at –ve m. (accept ≤)

(iv)   Integrate y to get area.
       $\rightarrow \left[\dfrac{x^4}{4} - x^3 - \dfrac{9x^2}{2} + kx \right]$ M1 A2,1   Attempt at integration. –1 each error.
       $\rightarrow 33.75$ when $x = 3$.   A1 [4]   co.

---

11    $f : x \mapsto k - x$
      $g : x \mapsto \dfrac{9}{x+2}$

(i) $k - x = \dfrac{9}{x+2}$
      $\rightarrow x^2 + (2-k)x + 9 - 2k = 0$ M1   Forming a quadratic equation.
      Use of $b^2 - 4ac$ M1   Use of $b^2 - 4ac$ on quadratic=0
      $\rightarrow a = 4$ or -8 DM1 A1   DM1 for solution. A1 both correct.
      $k = 4$, root is $\dfrac{-b}{2a} = 1$ M1   Any valid method.
      $k = -8$, root is -5.   A1 [6]   Both correct.

(ii) $fg(x) = 6 - \dfrac{9}{x+2}$ M1   Must be fg, not for gf.
      Equates and solves with 5   DM1   Reasonable algebra.
      $x = 7$ A1 [3]   co.

      [or $fg(x) = 5 \rightarrow g(x) = 1 \rightarrow x = 7$]   [g(x)=1 M1 → x DM1   x=7 A1]

(iii) $y = \dfrac{9}{x+2} \rightarrow x = \dfrac{9}{y} - 2$
      $g^{-1}(x) = \dfrac{9}{x} - 2$ or $\dfrac{9-2x}{x}$ M1 A1 [2]   Virtually correct algebra. Allow + for –. Correct and in terms of x.

DM1 for quadratic. Quadratic must be set to 0.
Factors. Attempt at two brackets. Each bracket set to 0 and solved.
Formula. Correct formula. Correct use, but allow for numerical slips in b^2 and $-4ac$.