

# Sequences and Series

## Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Sequences and Series
Booklet	Question Paper

**Time Allowed:** 125 minutes

**Score:** /104

**Percentage:** /100

**Grade Boundaries:**

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1 The coefficient of  $x^3$  in the expansion of  $(2 + ax)^5$  is 10 times the coefficient of  $x^2$  in  $\left(1 + \frac{ax}{3}\right)^4$ . Find  $a$ . [4]

2 An arithmetic progression has first term  $a$  and common difference  $d$ . The first, ninth and fourteenth terms are, respectively, the first three terms of a geometric progression with common ratio  $r$ , where  $r \neq 1$ .

(i) Find  $d$  in terms of  $a$  and show that  $r = \frac{5}{8}$ . [7]

(ii) Find the sum to infinity of the geometric progression in terms of  $a$ . [2]

3 It is given that  $x$ , 6 and  $x + 5$  are consecutive terms of a geometric progression.

(i) Show that  $x^2 + 5x - 36 = 0$  and find the possible values of  $x$ . [3]

(ii) Hence find the possible values of the common ratio. [2]

Furthermore,  $x$ , 6 and  $x + 5$  are the second, third and fourth terms of a geometric progression for which the sum to infinity exists.

(iii) Find the first term and the sum to infinity. [4]

4 An arithmetic progression has first term 5 and common difference 7.

(i) Find the value of the 10th term. [1]

(ii) Find the sum of the first 15 terms. [2]

The terms of the progression are given by  $x_1, x_2, x_3, \dots$ .

(iii) Evaluate  $\sum_{n=1}^{15} (2x_n + 1)$ . [3]

5 Find the coefficient of  $x^3$  in the expansion of  $(1 - 2x)^5$ . [4]

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- 6 (i) An arithmetic sequence has first term 3 and common difference 2. Find the twenty-first term of this sequence. [2]
- (ii) Find the sum to infinity of a geometric progression with first term 162 and second term 54. [3]
- (iii) A sequence is given by the recurrence relation  $u_1 = 3, u_{n+1} = 2 - u_n, n = 1, 2, 3, \dots$ . Find  $u_2, u_3, u_4, u_5$  and describe the behaviour of this sequence. [2]

7 Evaluate the following, giving your answers in exact form.

(i)  $\sum_{n=1}^{30} \frac{1}{n} - \sum_{n=2}^{29} \frac{1}{n}$ . [2]

(ii)  $\sum_{n=1}^{100} n \times (-1)^n$ . [2]

8 The first term of a geometric progression is 16 and the common ratio is 0.8.

(i) Calculate the sum of the first 12 terms. [3]

(ii) Find the sum to infinity. [2]

9 (i) Show that the first three terms in the expansion of  $(1 - 2x)^{\frac{1}{2}}$  are  $1 - x - \frac{1}{2}x^2$  and find the next term. [4]

(ii) State the range of values of  $x$  for which this expansion is valid. [1]

(iii) Hence show that the first four terms in the expansion of  $(2 + x)(1 - 2x)^{\frac{1}{2}}$  are  $2 - x + ax^2 + bx^3$  and state the values of  $a$  and  $b$ . [4]

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- 10** (i) An arithmetic sequence has first term 5 and fifth term 37.
- (a) Find an expression for  $u_n$ , the  $n$ th term of the sequence, in terms of  $n$ . [4]
- (b) Find an expression for  $S_n$ , the sum of the first  $n$  terms of this sequence, in terms of  $n$ . [2]
- (ii) Hence, or otherwise, calculate  $\sum_{n=5}^{25} (8n - 3)$ . [2]
- 11** (i) Find and simplify the first three terms in the expansion of  $(1 - 4a)^{\frac{1}{2}}$  in ascending powers of  $a$ , where  $|a| < \frac{1}{4}$ . [4]
- (ii) Hence show that the roots of the quadratic equation  $x^2 - x + a = 0$  are approximately  $1 - a - a^2$  and  $a + a^2$ , where  $a$  is small. [4]
- 12** An arithmetic progression has first term  $a$  and common difference  $d$ . The first ninth and fourteenth terms are, respectively, the first three terms of a geometric progression with common ratio  $r$ , where  $r \neq 1$ .
- (i) Find  $d$  in terms of  $a$  and show that  $r = \frac{5}{8}$ . [7]
- (ii) Find the sum to infinity of the geometric progression in terms of  $a$ . [2]
- 13** A geometric progression with common ratio  $r$  consists of positive terms. The sum of the first four terms is five times the sum of the first two terms.
- (i) Find an equation in  $r$  and deduce that  $r = 2$ . [3]
- (ii) Given that the fifth term is 192, find the value of the first term. [1]
- (iii) Find the smallest value of  $n$  such that the sum of the first  $n$  terms of the progression exceeds  $10^{64}$ . [3]

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14 Let  $f(x) = \frac{1+x^2}{\sqrt{4-3x}}$ .

(i) Obtain in ascending powers of  $x$  the first three terms in the expansion of  $\frac{1}{\sqrt{4-3x}}$  and state the values of  $x$  for which this expansion is valid. [5]

(ii) Hence obtain an approximation to  $f(x)$  in the form  $a + bx + cx^2$  where  $a$ ,  $b$  and  $c$  are constants. [2]

(iii) Use your approximation to estimate  $\int_0^{0.1} f(x) dx$ . [2]

15 An arithmetic progression has 13th term equal to 60 and 31st term equal to 141.

(i) Find the first term and common difference of the progression. [3]

A second arithmetic progression has first term 1.5 and common difference 3.

(ii) (a) Write down the first four terms of each progression. [1]

(b) Prove that the two progressions have an infinite number of terms in common. [2]