Trigonometry

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Trigonometry
Booklet	Question Paper

Time Allowed: **122 minutes**

Score: **/102**

Percentage: **/100**

Grade Boundaries:

1 0 **(i** Prove that $\cot\theta + \dfrac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$. [4

(ii) Hence solve the equation $\cot\left(\theta + \dfrac{\pi}{4}\right) + \dfrac{\sin\left(\theta + \dfrac{\pi}{4}\right)}{1+\cos\left(\theta + \dfrac{\pi}{4}\right)} = \dfrac{5}{2}$ for $0 \leqslant \theta \leqslant 2\pi$. [4

2 Sketch the curve with equation $y = \tan x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

On the same diagram, sketch the curve with equation $y = \tan^{-1} x$ for all x.

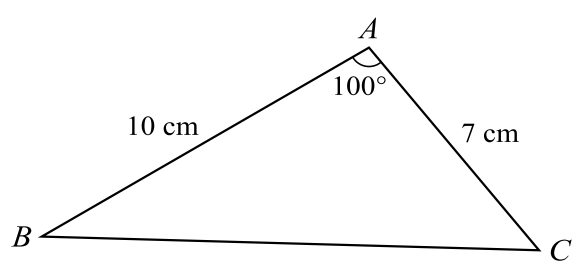
[3]

State the geometrical relationship between the curves.

3 **(i)** Use the identity $\tan 2x \equiv \dfrac{2\tan x}{1-\tan^2 x}$ to show that $\tan 4x \equiv \dfrac{4(1-\tan^2 x)\tan x}{1-6\tan^2 x+\tan^4 x}$. [6]

(ii) Hence, given that $x = \frac{1}{16}\pi$ is a root of the equation $\tan^4 x + p\tan^3 x - 6\tan^2 x - p\tan x + 1 = 0$ where p is a positive constant, f nd the value of p. [4]

4 The diagram shows the triangle ABC. $AB = 10$ cm, $AC = 7$ cm and angle $BAC = 100°$.



(i) Find the length BC. [2]

(ii) Find the area of the triangle ABC. [2]

5 Given that the angle θ is acute and $\cos\theta = \frac{3}{4}$ f nd, without using a calculator, the exact value of $\sin 2\theta$ and of $\cot\theta$. [5]

6 **(i)** Sketch the graph of $y = \cos 2x$ for $0 \leqslant x \leqslant 2\pi$. [2]

(ii) Describe the transformation which maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. [3]

7 **(i)** Show that $\sin\theta + \sqrt{3}\cos\theta$ can be expressed in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. State the values of R and α. [4]

(ii) Hence find the value of θ, where $0 < \theta < \pi$, such that $\sin\theta + \sqrt{3}\cos\theta = 0.8$. [4]

8 **(i)** Prove that $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$ and hence find an exact value for $\tan\left(\frac{3}{8}\pi\right)$. [6]

(ii) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi} (\operatorname{cosec} 2x - \cot 2x)^2 \, dx$. [6]

9 **(i)** Prove that

$$\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4(\cos^4\theta - \sin^4\theta)$$

and hence show that

$$\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4\cos 2\theta.$$ [5]

(ii) Hence solve the equation $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 2$ for $0° \leqslant \theta < 360°$. [4]

10 Sketch, on separate diagrams, the graphs of the following functions for $0 \leqslant x \leqslant 2\pi$ giving the coordinates of all points of intersection with the axes.

(i) $y = \sin x$. [1]

(ii) $y = \sin\left(x + \frac{1}{6}\pi\right)$. [2]

11 **(i)** Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and deduce that

$$\sin\theta + \sin 3\theta = 4\sin\theta\cos^2\theta.$$ [5]

(ii) Hence f nd the values of θ such that $0° < \theta < 180°$ that satisfy the equation

$$\cot^2\theta = \sin\theta + \sin 3\theta.$$ [4]

12 **(i)** On the same diagram, sketch the graphs of $y = 2\sec x$ and $y = 1 + 3\cos x$, for $0 \leqslant x \leqslant \pi$. [4]

(ii) Solve the equation $2\sec x = 1 + 3\cos x$, where $0 \leqslant x \leqslant \pi$. [5]

13 **(i)** Write down an identity for $\tan 2\theta$ in terms of $\tan\theta$ and use this result to show that

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}.$$ [4]

(ii) Given that $0 < \theta < \frac{1}{2}\pi$ and $\theta = \sin^{-1}\left(\frac{1}{\sqrt{10}}\right)$, show that $\tan 3\theta = \frac{13}{9}$. [3]

(iii) Show that the solutions of the equation

$$\tan(3\sin^{-1}x) = \tfrac{13}{9}$$

for $0 < x < 2\pi$ are

$$x = \frac{\sqrt{10}}{10} \qquad \text{and} \qquad x = \frac{\sqrt{10}\left(1 + 3\sqrt{3}\right)}{20}.$$ [4]

14 (i) Show that

$$\cos^4 x - \sin^4 x \equiv 2\cos^2 x - 1.$$ [2]

(ii) Hence f nd the solutions of

$$\cos^4 x - \sin^4 x = \cos x,$$

where $0° \leqslant x \leqslant 360°$. [4]