

Hypotesis test

Question Paper 8

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Hypotesis tests
Sub Topic	
Booklet	Question Paper 8

Time Allowed: 78 minutes

Score: /65

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

- 1 Jeevan thinks that a six-sided die is biased in favour of six. In order to test this, Jeevan throws the die 10 times. If the die shows a six on at least 4 throws out of 10, she will conclude that she is correct.
- (i) State appropriate null and alternative hypotheses. [1]
 - (ii) Calculate the probability of a Type I error. [3]
 - (iii) Explain what is meant by a Type II error in this situation. [1]
 - (iv) If the die is actually biased so that the probability of throwing a six is $\frac{1}{2}$, calculate the probability of a Type II error. [3]
- 2 At an election in 2010, 15% of voters in Bratfeld voted for the Renewal Party. One year later, a researcher asked 30 randomly selected voters in Bratfeld whether they would vote for the Renewal Party if there were an election next week. 2 of these 30 voters said that they would.
- (i) Use a binomial distribution to test, at the 4% significance level, the null hypothesis that there has been no change in the support for the Renewal Party in Bratfeld against the alternative hypothesis that there has been a decrease in support since the 2010 election. [4]
 - (ii) (a) Explain why the conclusion in part (i) cannot involve a Type I error. [1]
(b) State the circumstances in which the conclusion in part (i) would involve a Type II error. [1]
- 3 Metal bolts are produced in large numbers and have lengths which are normally distributed with mean 2.62 cm and standard deviation 0.30 cm.
- (i) Find the probability that a random sample of 45 bolts will have a mean length of more than 2.55 cm. [3]
 - (ii) The machine making these bolts is given an annual service. This may change the mean length of bolts produced but does not change the standard deviation. To test whether the mean has changed, a random sample of 30 bolts is taken and their lengths noted. The sample mean length is m cm. Find the set of values of m which result in rejection at the 10% significance level of the hypothesis that no change in the mean length has occurred. [4]

- 4 A hospital patient's white blood cell count has a Poisson distribution. Before undergoing treatment the patient had a mean white blood cell count of 5.2. After the treatment a random measurement of the patient's white blood cell count is made, and is used to test at the 10% significance level whether the mean white blood cell count has decreased.
- (i) State what is meant by a Type I error in the context of the question, and find the probability that the test results in a Type I error. [4]
 - (ii) Given that the measured value of the white blood cell count after the treatment is 2, carry out the test. [3]
 - (iii) Find the probability of a Type II error if the mean white blood cell count after the treatment is actually 4.1. [3]
- 5 At the 2009 election, $\frac{1}{3}$ of the voters in Chington voted for the Citizens Party. One year later, a researcher questioned 20 randomly selected voters in Chington. Exactly 3 of these 20 voters said that if there were an election next week they would vote for the Citizens Party. Test at the 2.5% significance level whether there is evidence of a decrease in support for the Citizens Party in Chington, since the 2009 election. [5]
- 6 Dipak carries out a test, at the 10% significance level, using a normal distribution. The null hypothesis is $\mu = 35$ and the alternative hypothesis is $\mu \neq 35$.
- (i) Is this a one-tail or a two-tail test? State briefly how you can tell. [1]
- Dipak finds that the value of the test statistic is $z = -1.750$.
- (ii) Explain what conclusion he should draw. [2]
 - (iii) This result is significant at the $\alpha\%$ level. Find the smallest possible value of α , correct to the nearest whole number. [2]

- 7 In a certain city it is necessary to pass a driving test in order to be allowed to drive a car. The probability of passing the driving test at the first attempt is 0.36 on average. A particular driving instructor claims that the probability of his pupils passing at the first attempt is higher than 0.36. A random sample of 8 of his pupils showed that 7 passed at the first attempt.
- (i) Carry out an appropriate hypothesis test to test the driving instructor's claim, using a significance level of 5%. [5]
 - (ii) In fact, most of this random sample happened to be careful and sensible drivers. State which type of error in the hypothesis test (Type I or Type II) could have been made in these circumstances and find the probability of this type of error when a sample of size 8 is used for the test. [4]
- 8 People who diet can expect to lose an average of 3 kg in a month. In a book, the authors claim that people who follow a new diet will lose an average of more than 3 kg in a month. The weight losses of the 180 people in a random sample who had followed the new diet for a month were noted. The mean was 3.3 kg and the standard deviation was 2.8 kg.
- (i) Test the authors' claim at the 5% significance level, stating your null and alternative hypotheses. [5]
 - (ii) State what is meant by a Type II error in words relating to the context of the test in part (i). [2]
- 9 When a guitar is played regularly, a string breaks on average once every 15 months. Broken strings occur at random times and independently of each other.
- (i) Show that the mean number of broken strings in a 5-year period is 4. [1]
- A guitar is fitted with a new type of string which, it is claimed, breaks less frequently. The number of broken strings of the new type was noted after a period of 5 years.
- (ii) The mean number of broken strings of the new type in a 5-year period is denoted by λ . Find the rejection region for a test at the 10% significance level when the null hypothesis $\lambda = 4$ is tested against the alternative hypothesis $\lambda < 4$. [4]
 - (iii) Hence calculate the probability of making a Type I error. [1]
- The number of broken guitar strings of the new type, in a 5-year period, was in fact 1.
- (iv) State, with a reason, whether there is evidence at the 10% significance level that guitar strings of the new type break less frequently. [2]