Hypotesis test

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Hypotesis tests
Sub Topic	
Booklet	Question Paper 5

Time Allowed: **63 minutes**

Score: **/52**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 In the past, the time taken by vehicles to drive along a particular stretch of road has had mean 12.4 minutes and standard deviation 2.1 minutes. Some new signs are installed and it is expected that the mean time will increase. In order to test whether this is the case, the mean time for a random sample of 50 vehicles is found. You may assume that the standard deviation is unchanged.

(i) The mean time for the sample of 50 vehicles is found to be 12.9 minutes. Test at the 2.5% signif cance level whether the population mean time has increased. [4]

(ii) State what is meant by a Type II error in this context. [2]

(iii) State what extra piece of information would be needed in order to f nd the probability of a Type II error. [1]

2 Cloth made at a certain factory has been found to have an average of 0.1 faults per square metre. Suki claims that the cloth made by her machine contains, on average, more than 0.1 faults per square metre. In a random sample of 5 m^2 of cloth from Suki's machine, it was found that there were 2 faults. Assuming that the number of faults per square metre has a Poisson distribution,

(i) state null and alternative hypotheses for a test of Suki's claim, [1]

(ii) test at the 10% signif cance level whether Suki's claim is justif ed. [4]

3 In the past, the fligh time, in hours, for a particular fligh has had mean 6.20 and standard deviation 0.80. Some new regulations are introduced. In order to test whether these new regulations have had any effect upon fligh times, the mean fligh time for a random sample of 40 of these flight is found.

(i) State what is meant by a Type I error in this context. [2]

(ii) The mean time for the sample of 40 f ights is found to be 5.98 hours. Assuming that the standard deviation of f ight times is still 0.80 hours, test at the 5% signif cance level whether the population mean f ight time has changed. [4]

(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in your answer to part **(ii)**. [2]

4 Marie claims that she can predict the winning horse at the local races. There are 8 horses in each race. Nadine thinks that Marie is just guessing, so she proposes a test. She asks Marie to predict the winners of the next 10 races and, if she is correct in 3 or more races, Nadine will accept Marie's claim.

(i) State suitable null and alternative hypotheses. [1]

(ii) Calculate the probability of a Type I error. [3]

(iii) State the signif cance level of the test. [1]

5 The lengths, in centimetres, of rods produced in a factory have mean μ and standard deviation 0.2.

The value of μ is supposed to be 250, but a manager claims that one machine is producing rods that are too long on average. A random sample of 40 rods from this machine is taken and the sample mean length is found to be 250.06 cm. Test at the 5% significanc level whether the manager's claim is justified [5]

6 Stephan is an athlete who competes in the high jump. In the past, Stephan has succeeded in 90% of jumps at a certain height. He suspects that his standard has recently fallen and he decides to carry out a hypothesis test to f nd out whether he is right. If he succeeds in fewer than 17 of his next 20 jumps at this height, he will conclude that his standard has fallen.

(i) Find the probability of a Type I error. [4]

(ii) In fact Stephan succeeds in 18 of his next 20 jumps. Which of the errors, Type I or Type II, is possible? Explain your answer. [2]

7 A researcher is investigating the actual lengths of time that patients spend with the doctor at their appointments. He plans to choose a sample of 12 appointments on a particular day.

(i) Which of the following methods is preferable, and why?

- Choose the first 12 appointments of the day.
- Choose 12 appointments evenly spaced throughout the day. [2]

Appointments are scheduled to last 10 minutes. The actual lengths of time, in minutes, that patients spend with the doctor may be assumed to have a normal distribution with mean μ and standard deviation 3.4. The researcher suspects that the actual time spent is more than 10 minutes on average. To test this suspicion, he recorded the actual times spent for a random sample of 12 appointments and carried out a hypothesis test at the 1% significance level.

(ii) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(iii) Given that the total length of time spent for the 12 appointments was 147 minutes, carry out the test. [5]

(iv) Give a reason why the Central Limit theorem was not needed in part **(iii)**. [1]

8 The weights, X kilograms, of rabbits in a certain area have population mean μ kg. A random sample of 100 rabbits from this area was taken and the weights are summarised by

$$\Sigma x = 165, \qquad \Sigma x^2 = 276.25.$$

Test at the 5% significance level the null hypothesis $H_0 : \mu = 1.6$ against the alternative hypothesis $H_1 : \mu \neq 1.6$. [6]