Population

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Sampling and estimation
Sub Topic	Population
Booklet	Question Paper 5

Time Allowed: **75 minutes**

Score: **/62**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 Each of a random sample of 15 students was asked how long they spent revising for an exam. The results, in minutes, were as follows.

50 70 80 60 65 110 10 70 75 60 65 45 50 70 50

Assume that the times for all students are normally distributed with mean μ minutes and standard deviation 12 minutes.

(i) Calculate a 92% confidence interval for μ. [4]

(ii) Explain what is meant by a 92% confidence interval for μ. [1]

(iii) Explain what is meant by saying that a sample is 'random'. [1]

2 The weights, in grams, of packets of sugar are distributed with mean μ and standard deviation 23. A random sample of 150 packets is taken. The mean weight of this sample is found to be 494 g. Calculate a 98% confidence interval for μ. [3]

3 A survey taken last year showed that the mean number of computers per household in Branley was 1.66. This year a random sample of 50 households in Branley answered a questionnaire with the following results.

Number of computers	0	1	2	3	4	> 4
Number of households	5	12	18	10	5	0

(i) Calculate unbiased estimates for the population mean and variance of the number of computers per household in Branley this year. [3]

(ii) Test at the 5% significance level whether the mean number of computers per household has changed since last year. [5]

(iii) Explain whether it is possible that a Type I error may have been made in the test in part **(ii)**. [1]

(iv) State what is meant by a Type II error in the context of the test in part **(ii)**, and give the set of values of the test statistic that could lead to a Type II error being made. [2]

4 In a sample of 50 students at Batlin college, 18 support the football club Real Madrid.

(i) Calculate an approximate 98% conf dence interval for the proportion of students at Batlin college who support Real Madrid. [4]

(ii) Give one condition for this to be a reliable result. [1]

5 Leaves from a certain type of tree have lengths that are distributed with standard deviation 3.2 cm. A random sample of 250 of these leaves is taken and the mean length of this sample is found to be 12.5 cm.

(i) Calculate a 99% conf dence interval for the population mean length. [3]

(ii) Write down the probability that the whole of a 99% conf dence interval will lie below the population mean. [1]

6 The lengths, x mm, of a random sample of 150 insects of a certain kind were found. The results are summarised by $\Sigma x = 7520$ and $\Sigma x^2 = 413\,540$.

(i) Calculate unbiased estimates of the population mean and variance of the lengths of insects of this kind. [3]

(ii) Using the values found in part **(i)**, calculate an estimate of the probability that the mean length of a further random sample of 80 insects of this kind is greater than 53 mm. [3]

7 **(a)** The time taken by a worker to complete a task was recorded for a random sample of 50 workers. The sample mean was 41.2 minutes and an unbiased estimate of the population variance was 32.6 minutes^2. Find a 95% conf dence interval for the mean time taken to complete the task. [3]

(b) The probability that an α% conf dence interval includes only values that are lower than the population mean is $\frac{1}{16}$. Find the value of α. [2]

8 In a random sample of 70 bars of Luxcleanse soap, 18 were found to be undersized.

(i) Calculate an approximate 90% confidence interval for the proportion of all bars of Luxcleanse soap that are undersized. [4]

(ii) Give a reason why your interval is only approximate. [1]

9 Previous records have shown that the number of cars entering Bampor on any day has mean 352 and variance 121.

(i) Find the probability that the mean number of cars entering Bampor during a random sample of 200 days is more than 354. [4]

(ii) State, with a reason, whether it was necessary to assume that the number of cars entering Bampor on any day has a normal distribution in order to find the probability in part **(i)**. [2]

(iii) It is thought that the population mean may recently have changed. The number of cars entering Bampor during the day was recorded for each of a random sample of 50 days and the sample mean was found to be 356. Assuming that the variance is unchanged, test at the 5% significance level whether the population mean is still 352. [4]

10 A random sample of n people were questioned about their internet use. 87 of them had a high-speed internet connection. A confidence interval for the population proportion having a high-speed internet connection is $0.1129 < p < 0.1771$.

(i) Write down the mid-point of this confidence interval and hence find the value of n. [3]

(ii) This interval is an α% confidence interval. Find α. [4]