Continuous random variables

Question Paper 7

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Continuous random variables
Sub Topic	
Booklet	Question Paper 7

Time Allowed: **80 minutes**

Score: **/66**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 A random variable X has probability density function given by

$$f(x) = \begin{cases} k(1-x) & -1 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2}$. [2]

(ii) Find $P\left(X > \frac{1}{2}\right)$. [1]

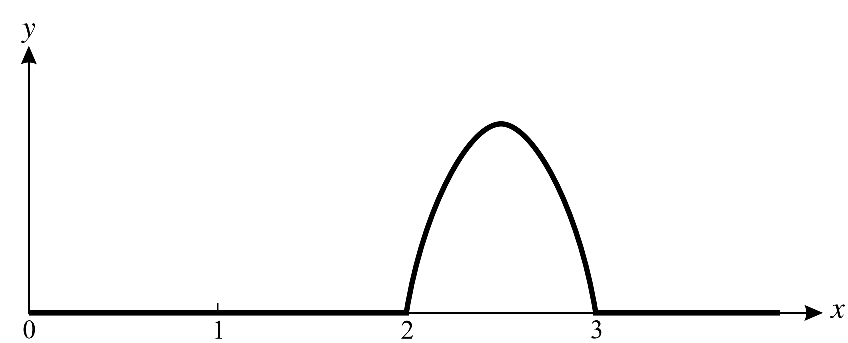
(iii) Find the mean of X. [3]

(iv) Find a such that $P(X < a) = \frac{1}{4}$. [3]

2 The distance travelled, in kilometres, by a Grippo brake pad before it needs to be replaced is modelled by $10\,000X$, where X is a random variable having the probability density function

$$f(x) = \begin{cases} -k(x^2 - 5x + 6) & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

The graph of $y = f(x)$ is shown in the diagram.

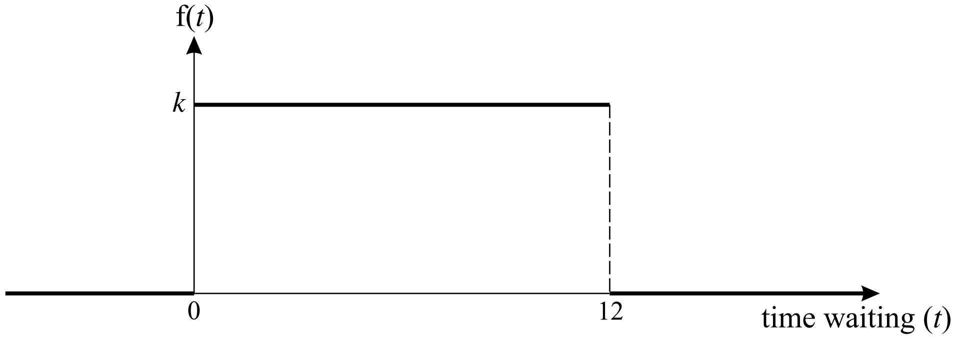


(i) Show that $k = 6$. [2]

(ii) State the value of $E(X)$ and f nd $\mathrm{Var}(X)$. [4]

(iii) Sami f ts four new Grippo brake pads on his car. Find the probability that at least one of these brake pads will need to be replaced after travelling less than 22 000 km. [3]

3



Fred arrives at random times on a station platform. The times in minutes he has to wait for the next train are modelled by the continuous random variable for which the probability density function f is shown above.

(i) State the value of k. [1]

(ii) Explain brief y what this graph tells you about the arrival times of trains. [1]

4 The random variable T denotes the time in seconds for which a f rework burns before exploding. The probability density function of T is given by

$$\mathrm{f}(t) = \begin{cases} ke^{0.2t} & 0 \leqslant t \leqslant 5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{5(\mathrm{e} - 1)}$. [3]

(ii) Sketch the probability density function. [2]

(iii) 80% of f reworks burn for longer than a certain time before they explode. Find this time. [3]

5 The time, in minutes, taken by volunteers to complete a task is modelled by the random variable X with probability density function given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{k}{x^4} & x \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = 3$. [2]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [6]

6 The time in minutes taken by candidates to answer a question in an examination has probability density function given by

$$f(t) = \begin{cases} k(6t - t^2) & 3 \leqslant t \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{18}$. [3]

(ii) Find the mean time. [3]

(iii) Find the probability that a candidate, chosen at random, takes longer than 5 minutes to answer the question. [2]

(iv) Is the upper quartile of the times greater than 5 minutes, equal to 5 minutes or less than 5 minutes? Give a reason for your answer. [2]

7 If Usha is stung by a bee she always develops an allergic reaction. The time taken in minutes for Usha to develop the reaction can be modelled using the probability density function given by

$$f(t) = \begin{cases} \dfrac{k}{t+1} & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{\ln 5}$. [4]

(ii) Find the probability that it takes more than 3 minutes for Usha to develop a reaction. [3]

(iii) Find the median time for Usha to develop a reaction. [3]

8 The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \frac{3}{4}(x^2 - 1) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Sketch the probability density function of X. [2]

(ii) Show that the mean, μ, of X is 1.6875. [3]

(iii) Show that the standard deviation, σ, of X is 0.2288, correct to 4 decimal places. [3]

(iv) Find $\mathrm{P}(1 \leqslant ; \leqslant \mu + \sigma)$. [3]