Continuous random variables
Question Paper 2

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Continuous random variables
Sub Topic	
Booklet	Question Paper 2

Time Allowed: **59 minutes**

Score: **/49**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 Darts are thrown at random at a circular board. The darts hit the board at distances X centimetres from the centre, where X is a random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{2}{a^2}x & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant.

  (i) Verify that f is a probability density function whatever the value of a. [3]

It is now given that $E(X) = 8$.

  (ii) Find the value of a. [3]

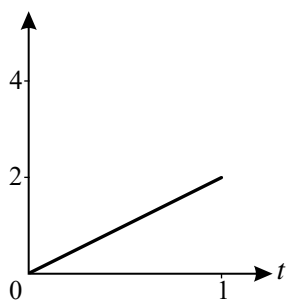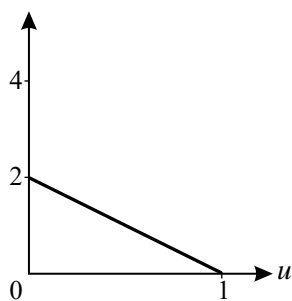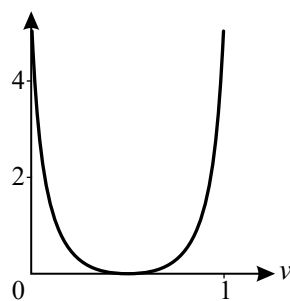 **(iii)** Find the probability that a dart lands more than $6\,\text{cm}$ from the centre of the board. [3]
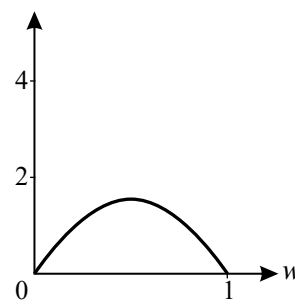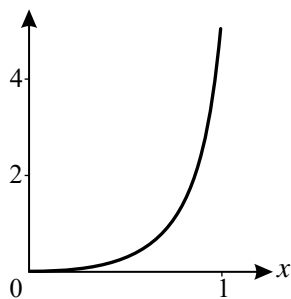
2

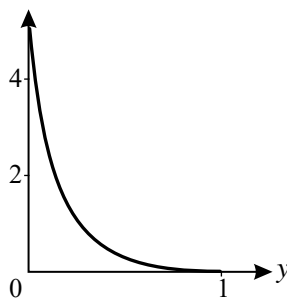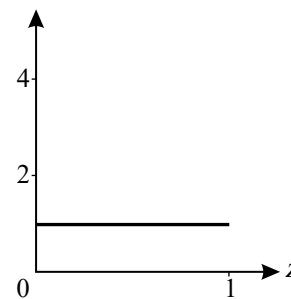Fig. 1   Fig. 2   Fig. 3   Fig. 4

Fig. 5   Fig. 6   Fig. 7

Each of the random variables T, U, V, W, X, Y and Z takes values between 0 and 1 only. Their probability density functions are shown in Figs 1 to 7 respectively.

  (i) (a)   Which of these variables has the largest median?   [1]

  (b)   Which of these variables has the largest standard deviation? Explain your answer.   [2]

 (ii) Use Fig. 2 to f nd $P(U < 0.5)$.   [2]

(iii) The probability density function of X is given by

$$f(x) = \begin{cases} ax^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and n are positive constants.

  (a)   Show that $a = n + 1$.   [3]

  (b)   Given that $E(X) = \frac{5}{6}$, f nd a and n.   [4]

3   The random variable X has probability density function given by

$$f(x) = \begin{cases} k\mathrm{e}^{-x} & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

    (i) Show that $k = \dfrac{\mathrm{e}}{\mathrm{e}-1}$.   [3]

    (ii) Find $\mathrm{E}(X)$ in terms of e.   [4]

4   People arrive randomly and independently at a supermarket checkout at an average rate of 2 people every 3 minutes.

    (i) Find the probability that exactly 4 people arrive in a 5-minute period.   [2]

At another checkout in the same supermarket, people arrive randomly and independently at an average rate of 1 person each minute.

    (ii) Find the probability that a total of fewer than 3 people arrive at the two checkouts in a 3-minute period.   [3]

5
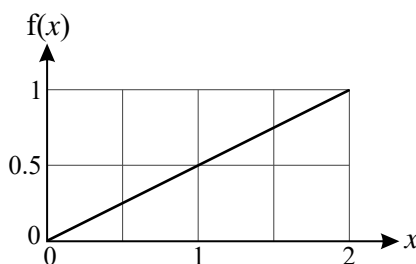
The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and 2 only.

    (i) Find $\mathrm{P}(1 < X < 1.5)$.   [2]

    (ii) Find the median of X.   [3]

    (iii) Find $\mathrm{E}(X)$.   [2]

6   A continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{6}x & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

   (i) Find $E(X)$. [3]

   (ii) Find the median of X. [3]

   (iii) Two independent values of X are chosen at random. Find the probability that both these values are greater than 3. [3]