

# Integration

## Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Integration
Booklet	Question Paper

**Time Allowed:** 108 minutes

**Score:** /90

**Percentage:** /100

**Grade Boundaries:**

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1 (a) Show that  $\int_0^2 \frac{x}{x^2 + 5} dx = \ln\left(\frac{3}{\sqrt{5}}\right)$ . [4]

(b) Find  $\int x\sqrt{x-2} dx$ . [4]

2 (i) Find  $\int (3x^2 - 4x + 8) dx$ . [3]

(ii) Hence find  $\int_1^3 (3x^2 - 4x + 8) dx$ . [2]

3 (i) Given that  $\frac{2x + 11}{(2x + 1)(x + 3)} \equiv \frac{A}{2x + 1} + \frac{B}{x + 3}$ , find the values of the constants  $A$  and  $B$ . [4]

(ii) Hence show that  $\int_0^2 \frac{2x + 11}{(2x + 1)(x + 3)} dx = \ln 15$ . [5]

4 (i) Use integration by parts to show that  $\int \ln x dx = x \ln x - x + c$ . [2]

(ii) Find

(a)  $\int (\ln x)^2 dx$ , [4]

(b)  $\int \frac{\ln(\ln x)}{x} dx$ . [5]

5 Find the exact value of  $\int_0^1 (e^x - x) dx$ . [4]

6 (i) (a) Find  $\int \frac{e^x}{1 + e^x} dx$ . [2]

(b) Hence evaluate  $\int_0^{\ln 3} \frac{e^x}{1 + e^x} dx$ , giving your answer in the form  $\ln k$ , where  $k$  is an integer. [3]

(ii) (a) Using the substitution  $u = 1 + e^x$ , find  $\int \left(\frac{e^x}{1 + e^x}\right)^2 dx$ . [5]

(b) Hence find the exact volume of the solid of revolution generated when the curve given by  $y = \frac{e^x}{1 + e^x}$ , between  $x = -\ln 3$  and  $x = \ln 3$ , is rotated through  $2\pi$  radians about the  $x$ -axis. [2]

7 A circle has equation  $x^2 + y^2 = 16$ . Find the volume generated when the region in the first quadrant which is bounded by the circle and the lines  $x = 1$  and  $x = 2$  is rotated through  $2\pi$  radians about the  $x$ -axis. [5]

8 Use integration by parts to find  $\int x \sin 3x dx$ . [5]

9 (i) Using the substitution  $u = x^2$ , or otherwise, find the numerical value of

$$\int_0^{\sqrt{\ln 4}} x e^{-\frac{1}{2}x^2} dx. [4]$$

(ii) Determine the exact coordinates of the stationary points of the curve  $y = x e^{-\frac{1}{2}x^2}$ . [4]

10 Using the substitution  $u = 1 + \sqrt{x}$ , or otherwise, find  $\int \frac{1}{1 + \sqrt{x}} dx$  giving your answer in terms of  $x$ . [5]

11 Find the exact value of

$$\int_1^4 (10x^{\frac{3}{2}} - 3x^{\frac{1}{2}}) dx. \quad [3]$$

12 (i) Show that

$$\int_1^a x^n \ln x dx = \frac{a^{n+1}}{(n+1)^2} ((n+1) \ln a - 1) + \frac{1}{(n+1)^2},$$

where  $n \neq -1$  and  $a > 1$ . [6]

(ii) (a) Determine the  $x$ -coordinate of the point of intersection of the curves  $y = x^2 \ln x$  and  $y = x \ln 2^x$ , where  $x > 0$ . [2]

(b) Find the exact value of the area of the region enclosed between these two curves, the line  $x = 1$  and their point of intersection. Express your answer in the form  $b + c \ln 2$ , where  $b$  and  $c$  are rational. [4]

(iii) The curve  $y = (x^3 \ln x)^{0.5}$ , for  $1 < x < e$ , is rotated through  $2\pi$  radians about the  $x$ -axis. Determine the value of the resulting volume of revolution, giving your answer correct to 4 significant figures. [3]