Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE Further Mathematics
AS Further Pure Mathematics FP1 Paper 8FM0_21

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$t = \tan\left(\frac{x}{2}\right),\ \ 5\sin x + 12\cos x = 2 \ \Rightarrow\ 7t^2 - 5t - 5 = 0$		
(a)	$\{5\sin x + 12\cos x = \}\ \ 5\left(\frac{2t}{1+t^2}\right) + 12\left(\frac{1-t^2}{1+t^2}\right)$	M1	1.1b
	$5\left(\frac{2t}{1+t^2}\right) + 12\left(\frac{1-t^2}{1+t^2}\right) = 2 \ \Rightarrow\ 5(2t) + 12(1-t^2) = 2(1+t^2)$	M1	1.1b
	$7t^2 - 5t - 5 = 0$ *	A1*	2.1
		(3)	
(b)	$t = \frac{5 \pm \sqrt{5^2 - 4(7)(-5)}}{2(7)}\ \ \left\{ = \frac{5 \pm \sqrt{165}}{14} = 1.2746...,\ -0.5603... \right\}$	M1	1.1a
	$\frac{x}{2} = \arctan\left(\frac{5+\sqrt{165}}{14}\right)$ or $\frac{x}{2} = \arctan\left(\frac{5-\sqrt{165}}{14}\right)$ **and** $\Rightarrow\ x = ...$	M1	3.1a
	$x =$ awrt 104° **or** $x =$ any answer in the range $[-58.6^{\circ}, -58^{\circ}]$	A1	1.1b
	$x = 103.8^{\circ}$ **and** $x = -58.5^{\circ}$	A1	1.1b
		(4)	
			(7 marks)

Notes

(a)	
M1:	Uses at least one of $\sin x = \frac{2t}{1+t^2}$ or $\cos x = \frac{1-t^2}{1+t^2}$ to express $5\sin x + 12\cos x$ in terms of t only
M1:	Uses both correct formula $\sin x = \frac{2t}{1+t^2}$ and $\cos x = \frac{1-t^2}{1+t^2}$ in $5\sin x + 12\cos x$, equates their expression to 2 and eliminates the fractions
A1*:	Collects terms to one side and simplifies to obtain the printed answer
(b)	
M1:	Selects a correct process (e.g. using the quadratic formula, completing the square or calculator approach) to solve $7t^2 - 5t - 5 = 0$
Note:	Allow 1st M1 for at least one of awrt 1.3 or awrt -0.6 or for a correct exact value of t
Note:	Do not allow an attempt at factorisation of $7t^2 - 5t - 5$ for the 1st M1
M1:	Adopts a correct ***applied*** strategy of taking arctan(their found t) and multiplying the result by 2 to obtain at least one value for x within the range $-180^{\circ} < x < 180^{\circ}$ (or in radians $-\pi < x < \pi$)
A1:	See scheme
A1:	For both 103.8 and -58.5
Note:	Give final A0 for extra solutions given within the range $-180^{\circ} < x < 180^{\circ}$
Note:	Ignore extra solutions outside the range $-180^{\circ} < x < 180^{\circ}$ for the final A mark
Note:	In degrees, $\frac{x}{2} = \{51.88...,\ -128.11...,\ -29.26...,\ 150.73...\}$
Note:	Working in radians gives $\frac{x}{2} = \{0.905...,\ -0.510...\} \Rightarrow x = \{1.81...,\ -1.02...\}$
Note:	Give 2nd M0 for $\frac{x}{2} = \{51.88...,\ -29.26...\}\ \Rightarrow\ x = \{\ 25.9,\ -14.6\ \}$

Question	Scheme	Marks	AOs
2	$\frac{d\theta}{dt} = -k(\theta - 20)$, k is a constant. $\theta_0 = 80$		
(a)	$\{$Two iterations from $t = 0$ to $t = 3 \Rightarrow \}$ $h = 1.5$		
	Uses $h = 1.5$, $\theta_0 = 80$, $k = 0.1$ (condone $k = -0.1$) in a complete strategy to find a numerical expression for $\theta_1 = ...$	M1	3.1b
	$\{\theta_0 = 80,\ k = 0.1 \Rightarrow\}$ $\left(\frac{d\theta}{dt}\right)_0 = -0.1(80 - 20)$ $\{= -6\}$	M1	3.4
	$\left\{\frac{\theta_1 - 80}{1.5} = -6 \Rightarrow\right\}$ $\theta_1 = 80 + (1.5)(-6)$	M1	1.1b
	$\theta_1 = 71$	A1	1.1b
	$\{\theta_1 = 71 \Rightarrow\}$ $\left(\frac{d\theta}{dt}\right)_1 = -0.1("71" - 20)$ $\{= -5.1\}$	M1	1.1b
	$\theta_2 = 71 + (1.5)(-5.1) = 63.35\ (^\circ C)$	A1	2.1
		(6)	
(b)	Decrease k to become a smaller positive value	B1	3.5c
		(1)	
		(7 marks)	

Notes

(a)	
M1:	See scheme
M1:	Uses the model to evaluate the initial value of $\frac{d\theta}{dt}$ using $k = 0.1$ (condone $k = -0.1$) and the initial condition $\theta_0 = 80$
M1:	Applies the approximation formula with $\theta_0 = 80$, $k = 0.1$ (condone $k = -0.1$) and their h to find a numerical expression for $\theta_1 = ...$
A1:	Finds the approximation for θ at 1.5 minutes as 71
M1:	Uses their 71 and $k = 0.1$ (condone $k = -0.1$) to find $\frac{d\theta}{dt}$
A1:	Applies the approximation formula again to give $63.35\ (^\circ C)$ or awrt $63(^\circ C)$
Note:	$h = 0.1 \Rightarrow \theta_1 = 79.4,\ \theta_2 = 78.806$; $h = 1 \Rightarrow \theta_1 = 74,\ \theta_2 = 68.6$; $h = 0.15 \Rightarrow \theta_1 = 79.1,\ \theta_2 = 78.2135$
(b)	
B1:	See scheme
Note:	Allow B1 for "the value of k should satisfy $0 < k < 0.1$"
Note:	Condone "the value of k would need to be decreased" for B1
Note:	Give B0 for "change k to become negative"

Question	Scheme	Marks	AOs
3	$\dfrac{x}{x^2-2x-3} \le \dfrac{1}{x+3}$		
	$\dfrac{x(x+3)-(x^2-2x-3)}{(x^2-2x-3)(x+3)} \le 0$ or $x(x-3)(x+1)(x+3)^2-(x-3)^2(x+1)^2(x+3) \le 0$ or $x(x^2-2x-3)(x+3)^2-(x^2-2x-3)^2(x+3) \le 0$	M1	2.1
	$\dfrac{5x+3}{(x-3)(x+3)(x+1)}\ \{\le 0\}$ or $(x-3)(x+1)(x+3)(5x+3)\ \{\le 0\}$	M1	1.1b
		A1	1.1b
	All three critical values $-3,\ 3,\ -1$	B1	1.1b
	Critical value $-\dfrac{3}{5}$	B1ft	1.1b
	$\{x\in\mathbb{R}: -3<x<-1\}\cup\left\{x\in\mathbb{R}: -\dfrac{3}{5}\le x<3\right\}$	M1	2.2a
		A1	2.5
		(7)	
			(7 marks)

Notes

M1:	Gathers terms on one side and puts over a common denominator, **or** multiplies by $(x+1)^2(x-3)^2(x+3)^2$ (or by the equivalent $(x^2-2x-3)^2(x+3)^2$) and gathers terms onto one side
M1:	Expands and simplifies fully the numerator **or** takes out a factor of $(x-3)(x+1)(x+3)$ (or the equivalent $(x^2-2x-3)(x+3)$) and then simplifies fully their remaining factor
A1:	$\dfrac{5x+3}{(x-3)(x+3)(x+1)}$ or $(x-3)(x+1)(x+3)(5x+3)$
B1:	Correct critical values of -3, 3 and -1 which can be implied, e.g. from their inequalities
B1ft:	Correct critical value of $-\dfrac{3}{5}$ which can be implied, e.g. from their inequalities
Note:	**B1ft:** You can follow through their fourth factor which is in the form $(ax+b)$, $a,b\ne 0$ to give C.V. $= -\dfrac{b}{a}$, if their fourth factor is not any of either $(x-3)$, $(x+3)$ or $(x+1)$
M1:	Deduces that 2 “inside” inequalities are required with critical values in ascending order
A1:	Exactly 2 correct intervals, condoning omission of the union symbol
Note:	Also accept, e.g. • $-3<x<-1,\ -\dfrac{3}{5}\le x<3$ • $(-3,\ -1),\ \left[-\dfrac{3}{5},\ 3\right)$ • $-1>x>-3,\ 3>x\ge -\dfrac{3}{5}$

Notes Continued	
Note:	Give 1st A0 for $(x^2-2x-3)(x+3)(5x+3)\ \{\le 0\}$ with no other working seen
Note:	Give 1st A1 (implied) for $(x^2-2x-3)(x+3)(5x+3)\ \{\le 0\}$ with $x=3,\ x=-1$ stated
Note:	Give 1st A0 for $\frac{5x+3}{(x^2-2x-3)(x+3)}\left\{\le 0\right\}$ with no other working seen
Note:	Give 1st A1 (implied) for $\frac{5x+3}{(x^2-2x-3)(x+3)}\left\{\le 0\right\}$ with $x=3,\ x=-1$ stated
Note:	Give 1st A0 for $\frac{5x+3}{x^3+x^2-9x-9}\left\{\le 0\right\}$ with no other working seen
Note:	Give 1st A1 (implied) for $\frac{5x+3}{x^3+x^2-9x-9}\left\{\le 0\right\}$ with $x=3,\ x=-1,\ x=-3$ stated
Note:	Allow special case final M1 for any of • $-3<x<-1$ (condoning closed inequalities or a mixture of open and closed inequalities) • $-\frac{3}{5}\le x<3$ (condoning closed inequalities or a mixture of open and closed inequalities) but ***do not allow*** M1 for any of • e.g. $-3<x<-1,\ -1<x\le -\frac{3}{5}$ ("continuing inequalities") • e.g. $-3<x<1,\ -\frac{3}{5}\le x<3$ ("overlapping inequalities")
	<u>Alternative Method</u> $x(x-3)(x+1)(x+3)^2\le (x-3)^2(x+1)^2(x+3)$ $x^5+4x^4-6x^3-36x^2-27x\le x^5-x^4-14x^3+6x^2+45x+27$ $5x^4+8x^3-42x^2-72x-27\le 0$
Note:	$5x^4+8x^3-42x^2-72x-27\le 0$ without any other working is M1M0A0
Note:	$5x^4+8x^3-42x^2-72x-27\le 0 \Rightarrow x=-3,\ -1,\ 3$ is M1M1A1B1
Note:	$5x^4+8x^3-42x^2-72x-27\le 0 \Rightarrow x=-3,\ -1,\ 3,\ -\frac{3}{5}$ is M1M1A1B1B1

Question	Scheme	Marks	AOs
4	$A(12,4,-1),\ B(10,15,-3),\ C(10,15,-3),\ D(2,2,-6)$		
(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\11\\-2\end{pmatrix},\ \overrightarrow{AC}=\begin{pmatrix}-7\\4\\6\end{pmatrix},\ \left\{\overrightarrow{BC}=\begin{pmatrix}-5\\-7\\8\end{pmatrix}\right\}$	M1	1.1b
	Area $=\frac{1}{2}\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 11 & -2\\-7 & 4 & 6\end{vmatrix}=\frac{1}{2}\left\lvert\begin{pmatrix}74\\26\\69\end{pmatrix}\right\rvert=\frac{1}{2}\sqrt{(74)^2+(26)^2+(69)^2}$	M1	1.1b
	$\{=52.23265...\}=52.2\ (\text{mm}^2)$ (1 dp) *	A1*	2.2a
		(3)	
(b)	Finds appropriate vectors to find the volume of *ABCD* and makes a complete attempt to find the volume of the tetrahedron	M1	3.1a
	e.g. $\left\lvert\begin{pmatrix}-10\\-2\\-5\end{pmatrix}\bullet\begin{pmatrix}74\\26\\69\end{pmatrix}\right\rvert=...$ or $\begin{vmatrix}-2 & 11 & -2\\-7 & 4 & 6\\-10 & -2 & -5\end{vmatrix}=...$	M1	1.1b
	$=\lvert-740-52-345\rvert$ or $\lvert-2(-8)-11(95)-2(54)\rvert$ $\{=1137\}$	A1	1.1b
	$V=\frac{1137}{6}\ (\text{mm}^3)$ $\left\{\text{or } \frac{379}{2} \text{ or } 189.5\right\}$	A1	1.1b
	Density $=\frac{0.5}{189.5}\times1000\ (\text{g cm}^{-3})$	M1	2.1
	$\{=2.638522427...\}=$ awrt $2.6\ (\text{g cm}^{-3})$	A1	1.1b
		(6)	
			(9 marks)

Notes

(a)	
M1:	Uses a correct method to find any 2 edges of triangle *ABC*
M1:	Complete process of taking the vector product between 2 edges of triangle *ABC*, applying Pythagoras and multiplying the result by 0.5
A1*:	Deduces the correct area of $52.2\ (\text{mm}^2)$. Condone awrt 52.2
Note:	Condone $\frac{1}{2}\lvert 74\mathbf{i}-26\mathbf{j}+69\mathbf{k}\rvert=\frac{1}{2}\sqrt{(74)^2+(-26)^2+(69)^2}=52.2$, o.e. for M1M1A1
Note:	As an alternative, $\frac{1}{2}\sqrt{129}\sqrt{101}\sin(66.2343...)=52.2$ (1 dp), where the angle has been found by applying the scalar product between $\overrightarrow{AB}$ and $\overrightarrow{AC}$
(b)	
M1:	See scheme
M1:	Uses appropriate vectors to in an attempt at the scalar triple product
A1:	Correct numerical expression for the scalar triple product (allow $\pm$)
A1:	Correct volume (in mm^3) (allow $\pm$)
M1:	A correct method for changing their units for their volume and for finding density
A1:	Obtains the correct density in g cm^{-3}. Allow awrt 2.6

Notes Continued	
(b)	
Note:	Using any of $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$ or $\overrightarrow{OD}$ in their scalar triple product is M0M0A0A0
Note:	Allow M1M1A0A0 for $V = \frac{1}{6}\left\|\begin{pmatrix}-10\\-2\\-5\end{pmatrix} \bullet \begin{pmatrix}74\\26\\69\end{pmatrix}\right\| = \frac{1}{6}\left\|-740\mathbf{i} - 52\mathbf{j} - 345\mathbf{k}\right\| = \frac{1}{6}\sqrt{(-740)^2 + (-52)^2 + (-345)^2}$ $= \frac{1}{6}(818.125296...) = 135.354216...$
Note:	Some vector product calculations for reference:
	$\left\|\overrightarrow{AD}.\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right\| = \begin{vmatrix}-10 & -2 & -5\\-2 & 11 & -2\\-7 & 4 & 6\end{vmatrix} = \left\|\begin{pmatrix}-10\\-2\\-5\end{pmatrix} \bullet \begin{pmatrix}74\\26\\69\end{pmatrix}\right\| = \left\|-740-52-345\right\| = 1137$
	$\left\|\overrightarrow{AB}.\left(\overrightarrow{AC} \times \overrightarrow{AD}\right)\right\| = \begin{vmatrix}-2 & 11 & -2\\-7 & 4 & 6\\-10 & -2 & -5\end{vmatrix} = \left\|\begin{pmatrix}-2\\11\\-2\end{pmatrix} \bullet \begin{pmatrix}-8\\-95\\54\end{pmatrix}\right\| = \left\|16-1045-108\right\| = 1137$
	$\left\|\overrightarrow{AC}.\left(\overrightarrow{AB} \times \overrightarrow{AD}\right)\right\| = \begin{vmatrix}-7 & 4 & 6\\-2 & 11 & -2\\-10 & -2 & -5\end{vmatrix} = \left\|\begin{pmatrix}-7\\4\\6\end{pmatrix} \bullet \begin{pmatrix}-59\\10\\114\end{pmatrix}\right\| = \left\|413+40+684\right\| = 1137$
Note:	Some candidates apply $\overrightarrow{AB} \times \overrightarrow{AC}$ incorrectly to give $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 11 & -2\\-7 & 4 & 6\end{vmatrix} = 74\mathbf{i} - 26\mathbf{j} + 69\mathbf{k}$ This leads to an incorrect $\left\|\overrightarrow{AD}.\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\right\| = \left\|\begin{pmatrix}-10\\-2\\-5\end{pmatrix} \bullet \begin{pmatrix}74\\-26\\69\end{pmatrix}\right\| = \left\|-740+52-345\right\| = 1033$

Question	Scheme	Marks	AOs
5	$H: xy = c^2,\ c \neq 0;\ P\left(cp, \frac{c}{p}\right),\ p \neq 0,$ lies on H		
(a)	**Either** $y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2}$ or $-\frac{c^2}{x^2}$ **or** $xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$ **or** $x = ct,\ y = \frac{c}{t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}.\frac{\mathrm{d}t}{\mathrm{d}x} = -\left(\frac{c}{t^2}\right)\left(\frac{1}{c}\right)$ **and** so, at $P\left(cp, \frac{c}{p}\right),\ m_T = -\frac{1}{p^2}$	M1	2.1
	So, $m_N = p^2$	A1	2.2a
	$y - \frac{c}{p} = "p^2"(x - cp)$ **or** $\frac{c}{p} = "p^2"(cp) + b \Rightarrow y = "p^2"x +$ their b	M1	1.1b
	correct algebra leading to $p^3x - py + c(1 - p^4) = 0$ *	A1*	2.1
		(4)	
(b)	$y = \frac{c^2}{x} \Rightarrow p^3x - p\frac{c^2}{x} + c(1 - p^4) = 0$ **or** $x = \frac{c^2}{y} \Rightarrow p^3\frac{c^2}{y} - py + c(1 - p^4) = 0$	M1	3.1a
	$p^3x^2 + c(1 - p^4)x - c^2p = 0$ **or** $py^2 - c(1 - p^4)y - c^2p^3 = 0$	A1	1.1b
	$(x - cp)(p^3x + c) = 0 \Rightarrow x = \ldots$ **or** $\left(y - \frac{c}{p}\right)\left(yp + cp^4\right) = 0 \Rightarrow y = \ldots$	M1	3.1a
	$x = -\frac{c}{p^3}$ **and** $y = -cp^3$ **or** $\{Q\}\left(-\frac{c}{p^3}, -cp^3\right)$	A1	1.1b
	Midpoint is $\left(\frac{1}{2}\left(cp - \frac{c}{p^3}\right), \frac{1}{2}\left(\frac{c}{p} - cp^3\right)\right)$	M1	1.1b
		A1	1.1b
		(6)	
(b) Alt 1	Let Q be $\left(cq, \frac{c}{q}\right)$, so $p^3cq - p\frac{c}{q} + c(1 - p^4) = 0$	M1	3.1a
	$p^3cq^2 - pc + c(1 - p^4)q = 0 \Rightarrow p^3q^2 + (1 - p^4)q - p = 0$	A1	1.1b
	$(q - p)(p^3q + 1) = 0 \Rightarrow q = \ldots$	M1	3.1a
	$\{Q\}\left(-\frac{c}{p^3}, -cp^3\right)$ **or** $x = -\frac{c}{p^3}$ **and** $y = -cp^3$	A1	1.1b
		(10 marks)	

Notes	
(a)	
M1:	Starts the process of establishing the gradient of the normal by differentiating $xy = c^2$ • to give $\frac{\mathrm{d}y}{\mathrm{d}x} = \pm k\,x^{-2}$; $k \neq 0$, or • by the product rule to give $\pm x\frac{\mathrm{d}y}{\mathrm{d}x} \pm y$, or • by parametric differentiation to give $\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}t}\right) \times \frac{1}{\left(\text{their } \frac{\mathrm{d}x}{\mathrm{d}t}\right)}$, condoning $t \equiv p$ **and** attempt to use $P\left(cp, \frac{c}{p}\right)$ to write down the gradient of the tangent to the curve in terms of p
A1:	Deduces the correct normal gradient p^2 from their tangent gradient which is found using calculus
M1:	Correct straight line method for an equation of a normal where $m_N\ (\neq m_T)$ is found by using calculus. **Note:** m_N must be a function of p for this mark
A1*:	Obtains $p^3x - py + c(1 - p^4) = 0$, by correct solution only
(b)	
M1:	Substitutes $y = \frac{c^2}{x}$ or $x = \frac{c^2}{y}$ into the printed equation to obtain an equation in x, c and p only **or** in y, c and p only
A1:	Obtains a 3TQ equation in x or a 3TQ equation in y
Note:	E.g. $p^3x^2 + cx - cp^4x = c^2p$ or $py^2 = cy - cp^4y + c^2p^3$ are acceptable for the 1st A mark
M1:	Recognises that one solution of the quadratic equation is already known and uses a correct factorisation method of solving a 3TQ to give either $x = \ldots$ or $y = \ldots$ Alternatively applies a correct quadratic formula method for solving a 3TQ
A1:	Correct coordinates for Q, which can be simplified or un-simplified Allow $x = -\frac{c}{p^3}$ and $y = -cp^3$
M1:	Uses $\left(cp, \frac{c}{p}\right)$ and their (x_Q, y_Q) and applies $\left(\frac{cp + \text{their } x_Q}{2}, \frac{\frac{c}{p} + \text{their } y_Q}{2}\right)$ to give (x_M, y_M), where x_M and y_M are both in terms of c and p only
A1:	Correct coordinates $\left(\frac{1}{2}\left(cp - \frac{c}{p^3}\right), \frac{1}{2}\left(\frac{c}{p} - cp^3\right)\right)$. Condone $\left(\frac{cp - \frac{c}{p^3}}{2}, \frac{\frac{c}{p} - cp^3}{2}\right)$
Note:	Condone $x = \frac{1}{2}\left(cp - \frac{c}{p^3}\right)$ **and** $y = \frac{1}{2}\left(\frac{c}{p} - cp^3\right)$ for the final A mark
Note:	You can apply isw after correctly stated coordinates for the midpoint of P and Q

Notes Continued	
(b)	
Alt 1	**(*for the first 4 marks*)**
M1:	Substitutes $x = cq$ and $y = \dfrac{c}{q}$ into the printed equation to obtain an equation in only *p*, *c* and *q*
A1:	Eliminates *c* and obtains a correct quadratic equation in *q*
Note:	E.g. $p^3q^2 + q - p^4q = p$ is acceptable for the 1st A mark
M1:	Recognises that one solution of the quadratic equation is already known and uses a correct factorisation method of solving a 3TQ to give $q = ...$ Alternatively applies a correct quadratic formula method for solving a 3TQ in *q*
A1:	Correct coordinates for *Q*, which can be simplified or un-simplified Allow $x = -\dfrac{c}{p^3}$ and $y = -cp^3$

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