Paper 4G: Decision Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Auxiliary equation: $\lambda^2 - \lambda - 1 = 0$ and attempt to solve	M1	1.1b
	$\lambda = \frac{1 \pm \sqrt{5}}{2} \Rightarrow u_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$, where A and B are arbitrary constants	M1 A1	1.1b 2.2a
		(3)	
(b)	Use given conditions to obtain two equations in A and B	M1	1.1b
	Attempt to solve to obtain an A and B	M1	1.1b
	$u_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\}$	A1	1.1b
		(3)	
			(6 marks)

Notes:

(a)

M1: writes down correct auxiliary equation and attempts to solve using either the formula or completing the square

M1: writes down the general solution in the form $u_n = A(\lambda_1)^n + B(\lambda_2)^n$ using their roots λ_1, λ_2 - dependent on the first M mark

A1: CAO – both lhs and rhs correct including defining *A* and *B* as (arbitrary) constants

(b)

M1: uses the correct initial conditions to write down two equations in *A* and *B* – for reference these equations are $A(1+\sqrt{5}) + B(1-\sqrt{5}) = 2$ and $A(1+\sqrt{5})^2 + B(1-\sqrt{5})^2 = 4$

M1: Attempts to solve these two equations (using a correct method but condone sign slips) to achieve a value for *A* and *B*

A1: CAO

Question	Scheme	Marks	AOs
2(a)	(table below)	B1	1.1b

	D	E	F	Available
A	25			25
B	13	24	18	55
C			20	20
Required	38	24	38	

(1)

(b)

Shadow costs		15	22	14
		D	E	F
0	A	X	-3	-5
-4	B	X	X	X
4	C	-8	-14	X

M1 2.1; A1 1.1b

	D	E	F
A			
B		$24-\theta$	$18+\theta$
C		θ	$20-\theta$

Entering CE, exiting CF

	D	E	F
A	25		
B	13	4	38
C		20	

M1 2.1; A1 1.1b

Shadow costs		15	22	14
		D	E	F
0	A	X	-3	-5
-4	B	X	X	X
-10	C	6	X	14

M1 1.1b; A1 1.1b

	D	E	F
A	$25-\theta$		θ
B	$13+\theta$		$38-\theta$
C			

	D	E	F
A			25
B	38	4	13
C		20	

Entering AF, exiting AD

M1 1.1b; A1 2.2a

Shadow costs		10	17	9
		D	E	F
0	A	5	2	X
1	B	X	X	X
-5	C	6	X	14

No negative IIs so optimal solution of £1085

M1 2.1; A1 1.1b; A1 2.4

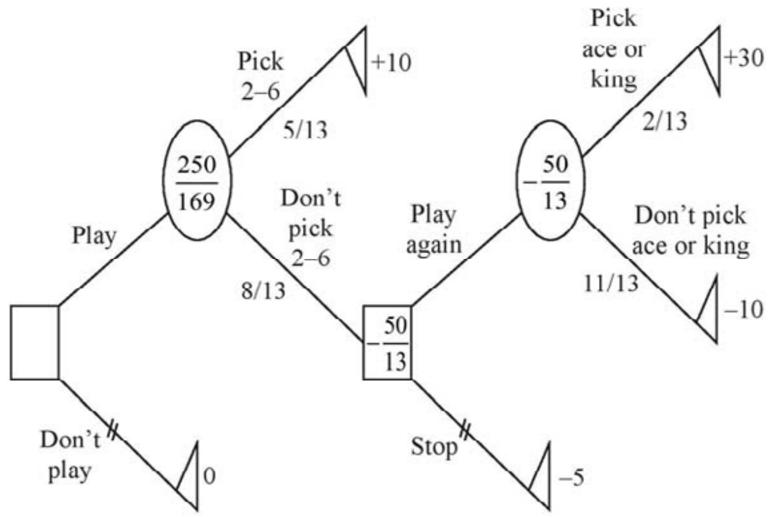
(11)

(12 marks)

Notes:	
B1:	CAO
M1:	Finding all 6 shadow costs and the 4 improvement indices for the correct 4 entries – candidates must clearly identify these two sets of results
A1:	Shadow costs and II CAO
M1:	A valid route, their most negative II chosen, only one empty square used, θ's balance
A1:	CAO
M1:	Finding all 6 shadow costs and the 4 improvement indices for the correct 4 entries
A1:	Shadow costs and II CAO
M1:	A valid route, their most negative II chosen, only one empty square used, θ's balance
A1:	CAO – including the deduction of all entering and exiting cells
M1:	Finding all 6 shadow costs and the 4 improvement indices for the correct 4 entries – this mark is depedent on all previous M marks which will therefore indicate a correct mathematical argument leading from the initial solution to the confirmation of the optimal solution
A1:	Shadow costs and II CAO
A1:	CSO including the correct reasoning that the solution is optimal because there are no negative IIs

Question	Scheme	Marks	AOs
3(a)	Subtract each entry from a constant (eg 40)	B1	2.4
		(1)	
(b)	e.g. $\begin{pmatrix} & P & Q & R & S \\ A & 5 & 5 & 4 & 2 \\ B & 9 & 2 & 6 & 0 \\ C & 2 & 8 & 4 & 1 \\ D & 1 & 7 & 1 & 4 \end{pmatrix}$	B1	1.1b
	Reducing row A by 2, no reduction for row B, reduce row C by 1 and row D by 1. No reduction of columns P, R and S, reduce column Q by 2.	B1	2.4
	$\begin{pmatrix} & P & Q & R & S \\ A & 3 & 3 & 2 & 0 \\ B & 9 & 2 & 6 & 0 \\ C & 1 & 7 & 3 & 0 \\ D & 0 & 6 & 0 & 3 \end{pmatrix}$ then $\begin{pmatrix} & P & Q & R & S \\ A & 3 & 1 & 2 & 0 \\ B & 9 & 0 & 6 & 0 \\ C & 1 & 5 & 3 & 0 \\ D & 0 & 4 & 0 & 3 \end{pmatrix}$	M1	2.1
		A1	1.1b
	Three lines required to cover the zeros hence solution is not optimal – augment by 1	B1	2.4
	$\begin{pmatrix} & P & Q & R & S \\ A & 2 & 1 & 1 & 0 \\ B & 8 & 0 & 5 & 0 \\ C & 0 & 5 & 2 & 0 \\ D & 0 & 5 & 0 & 4 \end{pmatrix}$	M1	2.1
	A – S, B – Q, C – P, D – R	A1	2.2a
		(7)	
(c)	$x_{ij} = \begin{cases} 1 \text{ if worker } i \text{ does task } j \\ 0 \quad \text{otherwise} \end{cases}$	B1	3.3
	Where $i \in \{A,B,C,D\}$ and $j \in \{P,Q,R,S\}$	B1	3.3
	e.g. Minimise $5x_{AP} + 5x_{AQ} + 4x_{AR} + 2x_{AS} + 9x_{BP} + 2x_{BQ} + 6x_{BR} +$ $2x_{CP} + 8x_{CQ} + 4x_{CR} + x_{CS} + x_{DP} + 7x_{DQ} + x_{DR} + 4x_{DS}$	B1	3.3
	Subject to:		
	$\sum x_{iP} = 1, \sum x_{iQ} = 1, \sum x_{iR} = 1, \sum x_{iS} = 1$	M1	3.3
	$\sum x_{Aj} = 1, \sum x_{Bj} = 1, \sum x_{Cj} = 1, \sum x_{Dj} = 1$	A1	3.3
		(5)	
			(13 marks)

Notes:	
(a)	
B1:	valid statement regarding converting a max. problem to a min. problem
(b)	
B1:	CAO
B1:	Correct statements regarding row and column reduction
M1:	Simplifying the initial matrix by reducing rows **and then** columns
A1:	CAO
B1:	Correct statements regarding both max. number of lines to cover zeros and augmentation
M1:	Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table)
A1:	CSO on final table (so must have scored all previous marks in this part) + deduction of the correct allocation
(c)	
B1:	possible values of x_{ij} defined
B1:	definine the set of values for i and j
B1:	Correct objective function and either 'minimise' or 'maximise' (dependent on if problem is defined in terms of original values or modified values)
M1:	at least four equations, unit coefficient and equal to 1
A1:	CAO (all eight equations)

Question	Scheme	Marks	AOs
4(a)		M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
		(6)	
(b)	EMV is 1.48 (tokens) per game (correct to 3 sf) Analysis: Play the game and if the player doesn’t pick a 2 – 6 on the first go then they should pick again	B1 B1	3.4 3.2a
		(2)	
			(8 marks)

Notes:

(a)

M1: Tree diagram with at least three end pay-offs, two decision nodes and two chance nodes

A1: Correct structure of tree diagram with each arc labelled correctly (including probabilities)

M1: At least three end-pay offs consistent with their stated probabilities; all five attempted

A1: CAO for end-pay offs

M1: End chance node follow through their end pay-offs and other chance/decision nodes completed

A1: CAO for decision and chance nodes including double lines through inferior options

(b)

B1: Correct EMV

B1: Correct analysis

Question	Scheme	Marks	AOs
5(a)	Column 2 <u>dominates</u> column 4	B1	2.5
	Because $2 > -2, 0 > -1$ and $3 > 2$	B1	2.4
		(2)	
(b)	Row minima: -2, -1, -1 max is -1 Column maxima: 4, 2, 3 min is 2	M1 A1	1.1b 1.1b
	Play safe is A plays 2 or 3 and B plays 2	A1	1.1b
	Row maximin (-1) $\neq$ Column minimax (2) so not stable	A1	2.4
		(4)	
(c)	e.g. $\begin{pmatrix} 4 & -2 & 3 \\ 3 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 6 & 0 & 5 \\ 5 & 1 & 4 \\ 1 & 4 & 2 \end{pmatrix}$	B1	1.1b
	Subject to $V - 6p_1 - 5p_2 - p_3 \leqslant 0$ $V - p_2 - 4p_3 \leqslant 0$ $V - 5p_1 - 4p_2 - 2p_3 \leqslant 0$	B1 B1	3.3 3.3
		(3)	
(d)	Substitute p values to obtain $V \leq \frac{19}{7}, \frac{19}{7}, \frac{20}{7} \therefore V = \frac{19}{7}$	M1	3.4
	Value of the game to player A $= \frac{19}{7} - 2 = \frac{5}{7}$	M1 A1	1.1b 1.1b
		(3)	
			(12 marks)

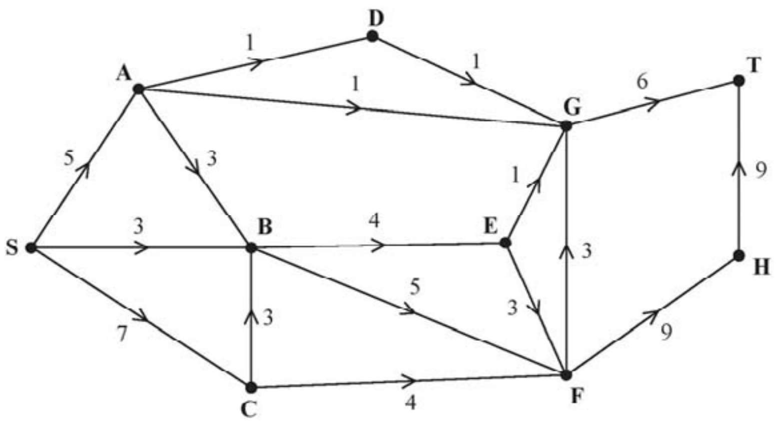
Notes:

(a)
B1: Correct statement – must include the word 'dominate'
B1: Correct inequalities – must be clear that all three inequalities must hold

(b)
M1: Attempt at row minima and column maxima – condone one error
A1: Correct max(row min) and min(col max)
A1: Correct play safe for both players
A1: Correct reasoning that the game is not stable (accept $-1 \neq 2$ + statement)

(c)
B1: Correct augmentation to make all entries non-negative
B1: At least one (of the three) equations **or** inequalities correct in V, p_1, p_2, p_3 (with all p_i terms in the constraint equations having correct signs)
B1: CAO - all three constraints correct involving V and p_i expressed as inequalities

(d)
M1: Substitute p values to obtain three values for V
M1: Their least value of V minus their augmented value
A1: CAO for the value of the game to player A

Question	Scheme	Marks	AOs
6(a)	$C_1 = 3+3+3+5+7=21$ $C_2 = 8+5+3+6-3=19$	B1 B1	1.1b 1.1b
		(2)	
(b)	e.g. the minimum flow out of the source S is at least 5 + 3 + 4 = 12 and the maximum flow into the sink T is 6 + 10 = 16	B1	2.4
		(1)	
(c)	The minimum flow into G is 1 + 1 + 1 + 3 = 6 but the maximum flow out of G is 6 therefore the arcs into G must be at their lower capacities	B1	2.4
		(1)	
(d)		M1 A1	3.1a 1.1b
		(2)	
(e)	Maximum flow is 15	B1	1.1b
	The minimum flow out of the source is 12 but the flow out of C is at least 3 + 4 = 7	B1	2.4
	Therefore the minimum flow through the network is 5 + 3 + 3 + 4 = 15 which is equal to the maximum flow	B1	2.2a
		(3)	
(f)	Increase the upper capacity of arc BF to at least 9 and therefore increase the flow in this arc to 9	B1	2.1
	Therefore increase the flow in FH and HT to 10	B1	2.4
	The flow in GT decreases to 5 and all other arcs are unchanged	B1	2.2a
		(3)	
			(12 marks)

Notes:	
(a)	
B1:	correct capacity for C_1
B1:	correct capacity for C_2
(b)	
B1:	correct statement regarding the min. flow out of the sink and max. flow into the sink
(c)	
B1:	correct statement regarding the flow into node G
(d)	
M1:	consistent flow pattern (≥ 12) throughout the network **-** so the flow into each node must equal the flow out of each node (and this flow must be greater than or equal to 12 but not necessarily the maximum flow of 15) - one number only on each arc
A1:	CAO
(e)	
B1:	CAO (for max. flow)
B1:	Consideration of both the min. flow from the source and the flow through node C
B1:	Completely correct argument that the max. flow = min. flow
(f)	
B1:	Correct argument regarding increasing the upper capacity of arc BF and hence the flow in that arc
B1:	Correct reasoning regarding increasing the flow in arcs FH and HT
B1:	Correct deduction that the flow in GT decreases to 5 and conclude that all other arcs are unchanged

Question	Scheme	Marks	AOs
7			

Stage	State	Action	Dest	Value	Marks	AOs
May	2	2	0	700 + 1500 = 2200*	M1	3.1b
(4)	1	3	0	350 + 1500 = 1850*	A1	1.1b
	0	4	0	1500 + 800 = 2300*		
April	2	1	0	700 + 1500 + 2300 = 4500		
(3)		2	1	700 + 1500 + 1850 = 4050*		
		3	2	700 + 1500 + 2200 = 4400	M1	3.1b
	1	2	0	350 + 1500 + 2300 = 4150	A1	1.1b
		3	1	350 + 1500 + 1850 = 3700*		
		4	2	350 + 1500 + 800 +2200 = 4850		
	0	3	0	1500 + 2300 = 3800*		
		4	1	1500 + 800 + 1850 = 4150		
		5	2	1500 + 800 + 2200 = 4500		
March	2	4	0	700 + 1500 +800 + 3800 = 6800		
(6)		5	1	700 + 1500 + 800 + 3700 = 6700*	M1	1.1b
	1	5	0	350 + 1500 + 800 + 3800 = 6450*	A1ft	1.1b
Feb	2	1	1	700 + 1500 + 6450 = 8650*		
(2)		2	2	700 + 1500 + 6700 = 8900		
	1	2	1	350 + 1500 + 6450 = 8300*		
		3	2	350 + 1500 + 6700 = 8550	M1	1.1b
	0	3	1	1500 + 6450 = 7950*	A1ft	1.1b
		4	2	1500 + 800 + 6700 = 9000		
Jan	0	3	0	1500 + 7950 = 9450*		
(3)		4	1	1500 + 800 + 8300 = 10600		
		5	2	1500 + 800 + 8650 = 10950	M1	1.1b
					A1	1.1b

Month	January	February	March	April	May	Marks	AOs
Number made	3	3	5	3	4	B1	1.1b

Scheme	Marks	AOs
Minimum production cost: £9450	B1	1.1b

(12 marks)

Notes:

All M marks – must bring optimal result from previous stage into calculations so for the second stage (April) if none of their 2200, 1850 or 2300 (the optimal results from May) are used then M0. Ignore extra rows. Condone and credit rows that have been crossed out if they can still be read. Must have right 'ingredients' (storage costs, additional space costs, overhead cost) at least once per stage. Must have values in two of the three colums (State, Action, Dest). If no working seen then the number stated in the Value column must be correct to imply the correct method has been used

1M1: First stage (May) completed. At least 3 rows, 'something' in each cell (but see M mark guidance above) including the correct structure (e.g. no value greater than 5 in the action column) in each of the first four columns

1A1: CAO for first stage.

2M1: Second stage (April) completed. At least 9 rows, something in each cell (see M mark guidance above) including the correct structure for the fifth (Value) column (e.g. bringing forward values from the previous stage)

2A1: CAO for second stage. No extra rows

3M1: Third stage (March) completed. At least 3 rows, something in each cell (see M mark guidance above)

3A1ft: CAO on the ft for third stage. No extra rows

4M1: Fourth stage (February) completed. At least 6 rows, something in each cell (see M mark guidance above)

4A1ft: CAO on the ft for fourth stage. No extra rows

5M1: Fifth stage (January) completed. At least 3 rows, something in each cell (see M mark guidance above)

5A1: CAO for the fifth stage. No extra rows

1B1: CAO – **but must have scored all previous M marks**

2B1: CAO – condone lack of units - **but must have scored all previous M marks**