Paper 2: Core Pure Mathematics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(i)	$\alpha+\beta+\gamma=8, \quad \alpha\beta+\beta\gamma+\gamma\alpha=28, \quad \alpha\beta\gamma=32$	B1	3.1a
	$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$	M1	1.1b
	$=\frac{7}{8}$	A1ft	1.1b
		(3)	
(ii)	$(\alpha+2)(\beta+2)(\gamma+2)=(\alpha\beta+2\alpha+2\beta+4)(\gamma+2)$	M1	1.1b
	$=\alpha\beta\gamma+2(\alpha\beta+\alpha\gamma+\beta\gamma)+4(\alpha+\beta+\gamma)+8$	A1	1.1b
	$=32+2(28)+4(8)+8=128$	A1	1.1b
		(3)	
	Alternative:		
	$(x-2)^3-8(x-2)^2+28(x-2)-32=0$	M1	1.1b
	$=\ldots-8+\ldots-32+\ldots-56-32=-128$	A1	1.1b
	$\therefore(\alpha+2)(\beta+2)(\gamma+2)=128$	A1	1.1b
		(3)	
(iii)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$	M1	3.1a
	$=8^2-2(28)=8$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:

(i)

B1: Identifies the correct values for all 3 expressions (can score anywhere)

M1: Uses a correct identity

A1ft: Correct value (follow through their 8, 28 and 32)

(ii)

M1: Attempts to expand

A1: Correct expansion

A1: Correct value

Alternative:

M1: Substitutes $x-2$ for x in the given cubic

A1: Calculates the correct constant term

A1: Changes sign and so obtains the correct value

(iii)

M1: Establishes the correct identity

A1ft: Correct value (follow through their 8, 28 and 32)

Question	Scheme	Marks	AOs
2(a)	$\begin{pmatrix}3\\-4\\2\end{pmatrix}\bullet\begin{pmatrix}6\\2\\12\end{pmatrix}=18-8+24$	M1	3.1a
	$d=\dfrac{18-8+24-5}{\sqrt{3^2+4^2+2^2}}$	M1	1.1b
	$=\sqrt{29}$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=\dots$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\dots$	M1	2.1
	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=0$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=0$ $\therefore\ -\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}3\\-4\\2\end{pmatrix}=-3+12+2$	M1	1.1b
	$\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}\cos\theta=11$ $\Rightarrow\cos\theta=\dfrac{11}{\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}}$	M1	2.1
	So angle between planes $\theta=52^\circ$ *	A1*	2.4
		(3)	
			(8 marks)

Notes:	
(a)	
M1:	Realises the need to and so attempts the scalar product between the normal and the position vector
M1:	Correct method for the perpendicular distance
A1:	Correct distance
(b)	
M1:	Recognises the need to calculate the scalar product between the given vector and both direction vectors
A1:	Obtains zero both times and makes a conclusion
(c)	
M1:	Calculates the scalar product between the two normal vectors
M1:	Applies the scalar product formula with their 11 to find a value for cos θ
A1*:	Identifies the correct angle by linking the angle between the normal and the angle between the planes

Question	Scheme	Marks	AOs
3(i)(a)	$\lvert\mathbf{M}\rvert = 2(1+2) - a(-1-1) + 4(2-1) = 0 \Rightarrow a = \ldots$	M1	2.3
	The matrix **M** has an inverse when $a \neq -5$	A1	1.1b
		(2)	
(b)	$\text{Minors}: \begin{pmatrix} 3 & -2 & 1 \\ -a-8 & 2 & a+4 \\ 4-a & -6 & -2-a \end{pmatrix}$ or $\text{Cofactors}: \begin{pmatrix} 3 & 2 & 1 \\ a+8 & 2 & -a-4 \\ 4-a & 6 & -2-a \end{pmatrix}$	B1	1.1b
	$\mathbf{M}^{-1} = \frac{1}{\lvert\mathbf{M}\rvert}\operatorname{adj}(\mathbf{M})$	M1	1.1b
	$\mathbf{M}^{-1} = \frac{1}{2a+10}\begin{pmatrix} 3 & a+8 & 4-a \\ 2 & 2 & 6 \\ 1 & -a-4 & -2-a \end{pmatrix}$ — 2 correct rows or columns. Follow through their det**M**	A1ft	1.1b
	All correct. Follow through their det**M**	A1ft	1.1b
		(4)	
(ii)	When $n = 1$, lhs $= \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$, rhs $= \begin{pmatrix} 3^1 & 0 \\ 3(3^1-1) & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$ So the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}$	M1	2.4
	$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & 0 \\ 3(3^k-1) & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}$	M1	2.1
	$= \begin{pmatrix} 3 \times 3^k & 0 \\ 3 \times 3(3^k-1)+6 & 1 \end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3(3^{k+1}-1) & 1 \end{pmatrix}$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4
		(6)	
			(12 marks)

Notes:	
(i)(a)	
M1:	Attempts determinant, equates to zero and attempts to solve for a in order to establish the restriction for a
A1:	Provides the correct condition for a if **M** has an inverse
(i)(b)	
B1:	A correct matrix of minors or cofactors
M1:	For a complete method for the inverse
A1ft:	Two correct rows following through their determinant
A1ft:	Fully correct inverse following through their determinant
(ii)	
B1:	Shows the statement is true for $n = 1$
M1:	Assumes the statement is true for $n = k$
M1:	Attempts to multiply the correct matrices
A1:	Correct matrix in terms of k
A1:	Correct matrix in terms of $k + 1$
A1:	Correct complete conclusion

Question	Scheme	Marks	AOs
4(a)	$z^n + z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta$	M1	2.1
	$= 2\cos n\theta$ *	A1*	1.1b
		(2)	
(b)	$\left(z + z^{-1}\right)^4 = 16\cos^4\theta$	B1	2.1
	$\left(z + z^{-1}\right)^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$	M1	2.1
	$= z^4 + z^{-4} + 4\left(z^2 + z^{-2}\right) + 6$	A1	1.1b
	$= 2\cos 4\theta + 4\left(2\cos 2\theta\right) + 6$	M1	2.1
	$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$*	A1*	1.1b
		(5)	
			(7 marks)

Notes:

(a)

M1: Identifies the correct form for z^n and z^{-n} and adds to progress to the printed answer

A1*: Achieves printed answer with no errors

(b)

B1: Begins the argument by using the correct index with the result from part (a)

M1: Realises the need to find the expansion of $\left(z + z^{-1}\right)^4$

A1: Terms correctly combined

M1: Links the expansion with the result in part (a)

A1*: Achieves printed answer with no errors

Question	Scheme	Marks	AOs
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sin x \cosh x + \cos x \sinh x$	M1	1.1a
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \cos x \cosh x + \sin x \sinh x + \cos x \cosh x - \sin x \sinh x$ $\left(= 2\cos x \cosh x\right)$	M1	1.1b
	$\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = 2\cos x \sinh x - 2\sin x \cosh x$	M1	1.1b
	$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = -4\sinh x \sin x = -4y$ *	A1*	2.1
		(4)	
(b)	$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)_0 = 2,\ \left(\frac{\mathrm{d}^6 y}{\mathrm{d}x^6}\right)_0 = -8,\ \left(\frac{\mathrm{d}^{10} y}{\mathrm{d}x^{10}}\right)_0 = 32$	B1	3.1a
	Uses $y = y_0 + xy_0' + \frac{x^2}{2!}y_0'' + \frac{x^3}{3!}y_0''' + \ldots$ with their values	M1	1.1b
	$= \frac{x^2}{2!}(2) + \frac{x^6}{6!}(-8) + \frac{x^{10}}{10!}(32)$	A1	1.1b
	$= x^2 - \frac{x^6}{90} + \frac{x^{10}}{113400}$	A1	1.1b
		(4)	
(c)	$2(-4)^{n-1}\frac{x^{4n-2}}{(4n-2)!}$	M1 A1	3.1a 2.2a
		(2)	
			(10 marks)

Notes:

(a)

M1: Realises the need to use the product rule and attempts first derivative

M1: Realises the need to use a second application of the product rule and attempts the second derivative

M1: Correct method for the third derivative

A1*: Obtains the correct 4th derivative and links this back to y

(b)

B1: Makes the connection with part **(a)** to establish the general pattern of derivatives and finds the correct non-zero values

M1: Correct attempt at Maclaurin series with their values

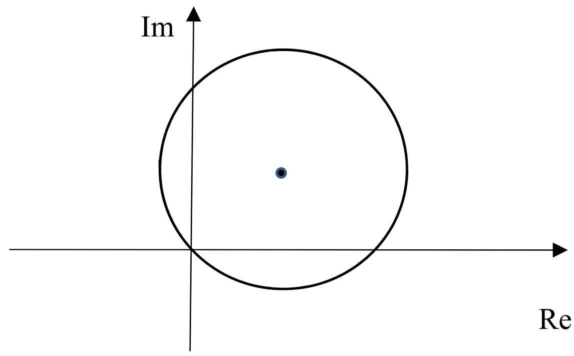
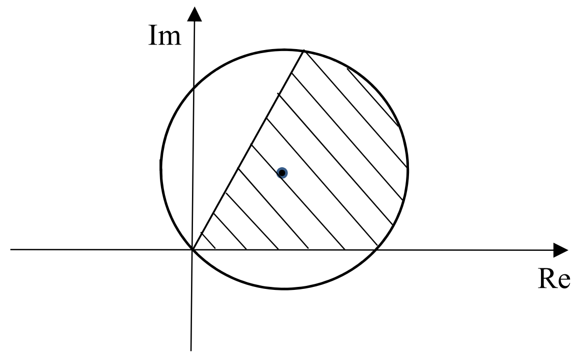
A1: Correct expression un-simplified

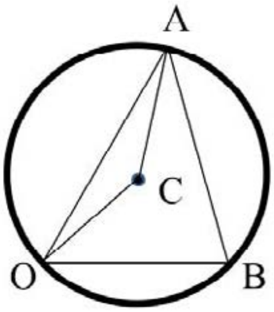
A1: Correct expression and simplified

(c)

M1: Generalising, dealing with signs, powers and factorials

A1: Correct expression

Question	Scheme	Marks	AOs
6(a)(i)		M1	1.1b
		A1	1.1b
(a)(ii)	$\lvert z-4-3\mathrm{i}\rvert=5 \Rightarrow \lvert x+\mathrm{i}y-4-3\mathrm{i}\rvert=5 \Rightarrow (x-4)^2+(y-3)^2=\ldots$	M1	2.1
	$(x-4)^2+(y-3)^2=25$ or any correct form	A1	1.1b
	$(r\cos\theta-4)^2+(r\sin\theta-3)^2=25$ $\Rightarrow r^2\cos^2\theta-8r\cos\theta+16+r^2\sin^2\theta-6r\sin\theta+9=25$ $\Rightarrow r^2-8r\cos\theta-6r\sin\theta=0$	M1	2.1
	$\therefore r=8\cos\theta+6\sin\theta$ *	A1*	2.2a
		(6)	
(b)(i)		B1	1.1b
		B1ft	1.1b
(b)(ii)	$A=\frac{1}{2}\int r^2\,\mathrm{d}\theta=\frac{1}{2}\int(8\cos\theta+6\sin\theta)^2\,\mathrm{d}\theta$ $=\frac{1}{2}\int(64\cos^2\theta+96\sin\theta\cos\theta+36\sin^2\theta)\mathrm{d}\theta$	M1	3.1a
	$=\frac{1}{2}\int(32(\cos 2\theta+1)+96\sin\theta\cos\theta+18(1-\cos 2\theta))\mathrm{d}\theta$	M1	1.1b
	$=\frac{1}{2}\int(14\cos 2\theta+50+48\sin 2\theta)\mathrm{d}\theta$	A1	1.1b
	$=\frac{1}{2}\left[7\sin 2\theta+50\theta-24\cos 2\theta\right]_0^{\frac{\pi}{3}}=\frac{1}{2}\left\{\left(\frac{7\sqrt{3}}{2}+\frac{50\pi}{3}+12\right)-(-24)\right\}$	M1	2.1
	$=\frac{7\sqrt{3}}{4}+\frac{25\pi}{3}+18$	A1	1.1b
		(7)	

Question	Scheme	Marks	AOs
			
	Alternative: Candidates may take a geometric approach e.g. by finding sector + 2 triangles		
	Angle $ACB = \left(\frac{2\pi}{3}\right)$ so area sector $ACB = \frac{1}{2}(5)^2\frac{2\pi}{3}$ Area of triangle $OCB = \frac{1}{2}\times 8\times 3$	M1	3.1a
	Sector area ACB + triangle area $OCB = \frac{25\pi}{3} + 12$	A1	1.1b
	Area of triangle OAC: Angle $ACO = 2\pi - \frac{2\pi}{3} - \cos^{-1}\left(\frac{5^2+5^2-8^2}{2\times5\times5}\right)$ so area $OAC = \frac{1}{2}(5)^2\sin\left(\frac{4\pi}{3} - \cos^{-1}\left(\frac{-7}{25}\right)\right)$	M1	1.1b
	$= \frac{25}{2}\left(\sin\frac{4\pi}{3}\cos\left(\cos^{-1}\left(\frac{-7}{25}\right)\right) - \cos\frac{4\pi}{3}\sin\left(\cos^{-1}\left(\frac{-7}{25}\right)\right)\right)$ $= \frac{25}{2}\left(\left(\frac{7\sqrt{3}}{50}\right) + \frac{1}{2}\sqrt{1-\left(\frac{7}{25}\right)^2}\right) = \frac{7\sqrt{3}}{4} + 6$ Total area $= \frac{25\pi}{3} + \frac{1}{2}\times8\times3 + 6 + \frac{7\sqrt{3}}{4}$	M1	2.1
	$= \frac{7\sqrt{3}}{4} + \frac{25\pi}{3} + 18$	A1	1.1b
		(13 marks)	

Notes:
(a)(i) **M1:** Draws a circle which passes through the origin **A1:** Fully correct diagram
(a)(ii) **M1:** Uses $z = x + iy$ in the given equation and uses modulus to find equation in x and y only **A1:** Correct equation in terms of x and y in any form – may be in terms of r and θ **M1:** Introduces polar form, expands and uses $\cos^2\theta + \sin^2\theta = 1$ leading to a polar equation **A1*:** Deduces the given equation (ignore any reference to $r = 0$ which gives a point on the curve)
(b)(i) **B1:** Correct pair of rays added to their diagram **B1ft:** Area between their pair of rays and inside their circle from (a) shaded, as long as there is an intersection
(b)(ii) **M1:** Selects an appropriate method by linking the diagram to the polar curve in (a), evidenced by use of the polar area formula **M1:** Uses double angle identities **A1:** Correct integral **M1:** Integrates and applies limits **A1:** Correct area
(b)(ii) Alternative: **M1:** Selects an appropriate method by finding angle ACB and area of sector ACB and finds area of triangle OCB to make progress towards finding the required area **A1:** Correct combined area of sector ACB + triangle OCB **M1:** Starts the process of finding the area of triangle OAC by calculating angle ACO and attempts area of triangle OAC **M1:** Uses the addition formula to find the exact area of triangle OAC and employs a full correct method to find the area of the shaded region **A1:** Correct area

Question	Scheme	Marks	AOs
7(a)	$r=10\frac{\mathrm{d}f}{\mathrm{d}t}-2f \Rightarrow \frac{\mathrm{d}r}{\mathrm{d}t}=10\frac{\mathrm{d}^2 f}{\mathrm{d}t^2}-2\frac{\mathrm{d}f}{\mathrm{d}t}$	M1	2.1
	$10\frac{\mathrm{d}^2 f}{\mathrm{d}t^2}-2\frac{\mathrm{d}f}{\mathrm{d}t}=-0.2f+0.4\left(10\frac{\mathrm{d}f}{\mathrm{d}t}-2f\right)$	M1	2.1
	$\frac{\mathrm{d}^2 f}{\mathrm{d}t^2}-0.6\frac{\mathrm{d}f}{\mathrm{d}t}+0.1f=0$*	A1*	1.1b
		(3)	
(b)	$m^2-0.6m+0.1=0 \Rightarrow m=\frac{0.6\pm\sqrt{0.6^2-4\times 0.1}}{2}$	M1	3.4
	$m=0.3\pm 0.1\mathrm{i}$	A1	1.1b
	$f=\mathrm{e}^{\alpha t}\left(A\cos\beta t+B\sin\beta t\right)$	M1	3.4
	$f=\mathrm{e}^{0.3t}\left(A\cos 0.1t+B\sin 0.1t\right)$	A1	1.1b
		(4)	
(c)	$\frac{\mathrm{d}f}{\mathrm{d}t}=0.3\mathrm{e}^{0.3t}\left(A\cos 0.1t+B\sin 0.1t\right)+0.1\mathrm{e}^{0.3t}\left(B\cos 0.1t-A\sin 0.1t\right)$	M1	3.4
	$r=10\frac{\mathrm{d}f}{\mathrm{d}t}-2f$ $=\mathrm{e}^{0.3t}\left((3A+B)\cos 0.1t+(3B-A)\sin 0.1t\right)-2\mathrm{e}^{0.3t}\left(A\cos 0.1t+B\sin 0.1t\right)$	M1	3.4
	$r=\mathrm{e}^{0.3t}\left((A+B)\cos 0.1t+(B-A)\sin 0.1t\right)$	A1	1.1b
		(3)	
(d)(i)	$t=0, f=6 \Rightarrow A=6$	M1	3.1b
	$t=0, r=20 \Rightarrow B=14$	M1	3.3
	$r=\mathrm{e}^{0.3t}\left(20\cos 0.1t+8\sin 0.1t\right)=0$	M1	3.1b
	$\tan 0.1t=-2.5$	A1	1.1b
	2019	A1	3.2a
(d)(ii)	3750 foxes	B1	3.4
(d)(iii)	e.g. the model predicts a large number of foxes are on the island when the rabbits have died out and this may not be sensible	B1	3.5a
		(7)	
			(17 marks)

Notes:
(a) **M1:** Attempts to differentiate the first equation with respect to t **M1:** Proceeds to the printed answer by substituting into the second equation **A1*:** Achieves the printed answer with no errors
(b) **M1:** Uses the model to form and solve the auxiliary equation **A1:** Correct values for m **M1:** Uses the model to form the CF **A1:** Correct CF
(c) **M1:** Differentiates the expression for the number of foxes **M1:** Uses this result to find an expression for the number of rabbits **A1:** Correct equation
(d)(i) **M1:** Realises the need to use the initial conditions in the model for the number of foxes **M1:** Realises the need to use the initial conditions in the model for the number of rabbits to find both unknown constants **M1:** Obtains an expression for r in terms of t and sets = 0 **A1:** Rearranges and obtains a correct value for tan **A1:** Identifies the correct year
(d)(ii) **B1:** Correct number of foxes
(d)(iii) **B1:** Makes a suitable comment on the outcome of the model