IB Maths: Analysis & Approaches SL & HL

Formula sheet for use during the IB Maths course & examinations

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Prior Learning SL & HL

Area of a parallelogram	$A = bh$	b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$	b is the base, h is the height
Area of a trapezium	$A = \frac{1}{2}(a+b)h$	a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$	r is the radius
Circumference of a circle	$C = 2\pi r$	r is the radius
Volume of a cuboid	$V = lwh$	l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$	r is the radius, h is the height
Volume of a prism	$V = Ah$	A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$	r is the radius, h is the height
Distance between two points (x_1, y_1) & (x_2, y_2)	$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$	
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) & (x_2, y_2)	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$	

Topic 1: Number & Algebra – SL & HL

The nth term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1+(n-1)d)$; $S_n = \frac{n}{2}(u_1+u_n)$
The nth term of a geometric sequence	$u_n = u_1 r^{n-1}$
The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n-1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
Compound interest	$FV = PV \times \left(1+\frac{r}{100k}\right)^{kn}$ FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, $r\%$ is the nominal annual rate of interest
Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$ $a>0, b>0, a\neq 1$
Exponents & logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m\log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}$, $\lvert r\rvert < 1$
Binomial theorem	$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n$ ${}^nC_r = \frac{n!}{r!(n-r)!}$

Topic 1: Number & Algebra – HL only

Combinations	${}^nC_r = \frac{n!}{r!(n-r)!}$
Permutations	${}^nP_r = \frac{n!}{(n-r)!}$
Complex numbers	$z = a + bi$
Modulus-argument (polar) & exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\text{cis}\,\theta$
De Moivre's theorem	$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta} = r^n\,\text{cis}\,n\theta$

Topic 2: Functions – SL & HL

Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$
Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, $a \neq 0$
Discriminant	$\Delta = b^2 - 4ac$
Exponential & logarithmic functions	$a^x = e^{x\ln a}$; $\log_a a^x = x = a^{\log_a x}$ $a, x > 0, a \neq 1$

Topic 2: Functions – HL only

Sum & product of the roots of polynomial equations of the form $\sum_{r=0}^{n} a_r x^r = 0$	Sum is $\frac{-a_{n-1}}{a_n}$; product is $\frac{(-1)^n a_0}{a_n}$

Make sure you check that your Graphical Display Calculator is on the list of approved calculators released by the exam board

Level	Paper	Length	Marks
SL	1	90 mins	80
SL	2	90 mins	80
HL	1	2 hours	110
HL	2	2 hours	110
HL	3	1 hour	55

Topic 3: Geometry & Trigonometry – SL & HL

Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) & (x_2, y_2, z_2)	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$
Volume of a right-pyramid	$V = \frac{1}{3}Ah$, A is the area of the base, h is the height
Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$ r is the radius, h is the height
Area of the curved surface of a cone	$A = \pi rl$ r is the radius, l is the slant height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$ r is the radius
Surface area of a sphere	$A = 4\pi r^2$ r is the radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2+b^2-c^2}{2ab}$
Area of a triangle	$A = \frac{1}{2}ab\sin C$
Length of an arc	$l = r\theta$ r is the radius θ is the angle measured in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$ r is the radius, θ is the angle measured in radians
Identity for $\tan\theta$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle identities	$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

Topic 3: Geometry & Trigonometry – HL only

Reciprocal trigonometric identities	$\sec\theta = \frac{1}{\cos\theta}$ $\text{cosec}\,\theta = \frac{1}{\sin\theta}$
Pythagorean identities	$1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$
Compound angle identities	$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$ $\tan(A\pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$
Double angle identity for tan	$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$
Magnitude of a vector	$\lvert \boldsymbol{v}\rvert = \sqrt{v_1^2+v_2^2+v_3^2}$ $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
Scalar product	$\boldsymbol{v}\cdot\boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3$ $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\boldsymbol{v}\cdot\boldsymbol{w} = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\cos\theta$, where θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Angle between two vectors	$\cos\theta = \frac{v_1w_1+v_2w_2+v_3w_3}{\lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert}$
Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$
Parametric form of the equation of a line	$x = x_0 + \lambda l$, $y = y_0 + \lambda m$, $z = z_0 + \lambda n$
Cartesian equations of a line	$\frac{x-x_0}{l} = \frac{y-y_0}{m} = \frac{z-z_0}{n}$
Vector product	$\boldsymbol{v}\times\boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$, $\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$ $\lvert\boldsymbol{v}\times\boldsymbol{w}\rvert = \lvert\boldsymbol{v}\rvert\lvert\boldsymbol{w}\rvert\sin\theta$ θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Area of a parallelogram	$A = \lvert\boldsymbol{v}\times\boldsymbol{w}\rvert$ $\boldsymbol{v}$ and $\boldsymbol{w}$ form two adjacent sides of a parallelogram
Vector equation of a plane	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$
Equation of a plane (using the normal vector)	$\boldsymbol{r}\cdot\boldsymbol{n} = \boldsymbol{a}\cdot\boldsymbol{n}$
Cartesian equation of a plane	$ax + by + cz = d$

Topic 4: Statistics & Probability – SL & HL

Interquartile range	$\text{IQR} = Q_3 - Q_1$
Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ $n = \sum_{i=1}^{k} f_i$
Probability of an event A	$P(A) = \frac{n(A)}{n(U)}$
Complementary events	$P(A) + P(A') = 1$
Combined events	$P(A\cup B) = P(A) + P(B) - P(A\cap B)$
Mutually exclusive events	$P(A\cup B) = P(A) + P(B)$
Conditional probability	$P(A\mid B) = \frac{P(A\cap B)}{P(B)}$
Independent events	$P(A\cap B) = P(A)\,P(B)$
Expected value of a discrete random variable X	$E(X) = \sum x\,P(X = x)$
Binomial distribution $X \sim B(n, p)$	
Mean	$E(X) = np$
Variance	$\text{Var}(X) = np(1-p)$
Standardized normal variable	$z = \frac{x-\mu}{\sigma}$

Topic 4: Statistics & Probability – HL only

Bayes' theorem	$P(B\mid A) = \frac{P(B)\,P(A\mid B)}{P(B)\,P(A\mid B) + P(B')\,P(A\mid B')}$ $P(B_i\mid A) = \frac{P(B_i)\,P(A\mid B_i)}{P(B_1)P(A\mid B_1) + P(B_2)P(A\mid B_2) + P(B_3)P(A\mid B_3)}$
Variance σ^2	$\sigma^2 = \frac{\sum_{i=1}^{k} f_i(x_i-\mu)^2}{n} = \frac{\sum_{i=1}^{k} f_i x_i^2}{n} - \mu^2$
Standard deviation σ	$\sigma = \sqrt{\frac{\sum_{i=1}^{k} f_i(x_i-\mu)^2}{n}}$
Linear transformation of a single random variable	$E(aX + b) = aE(X) + b$ $\text{Var}(aX+b) = a^2\,\text{Var}(X)$
Expected value of a continuous random variable X	$E(X) = \mu = \int_{-\infty}^{\infty} x\,f(x)\,dx$
Variance	$\text{Var}(X) = E(X-\mu)^2 = E(X^2) - [E(X)]^2$
Variance of a discrete random variable X	$\text{Var}(X) = \sum(x-\mu)^2\,P(X=x) = \sum x^2\,P(X=x) - \mu^2$
Variance of a continuous random variable X	$\text{Var}(X) = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

Topic 5: Calculus – SL & HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$
Area between a curve $y = f(x)$ & the x-axis, where $f(x) > 0$	$A = \int_a^b y\,dx$
Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$, where $u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$
Product rule	$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Acceleration	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
Distance travelled from t_1 to t_2	distance $= \int_{t_1}^{t_2} \lvert v(t)\rvert\,dt$
Displacement from t_1 to t_2	displacement $= \int_{t_1}^{t_2} v(t)\,dt$
Standard integrals	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$ $\int \sin x\,dx = -\cos x + C$ $\int \cos x\,dx = \sin x + C$ $\int e^x\,dx = e^x + C$
Area of region enclosed by a curve and x-axis	$A = \int_a^b \lvert y\rvert\,dx$

Topic 5: Calculus – HL only

Derivative of $f(x)$ from first principles	$y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$
Standard derivatives	
$\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$
$\sec x$	$f(x) = \sec x \Rightarrow f'(x) = \sec x\tan x$
$\text{cosec}\,x$	$f(x) = \text{cosec}\,x \Rightarrow f'(x) = -\text{cosec}\,x\cot x$
$\cot x$	$f(x) = \cot x \Rightarrow f'(x) = -\text{cosec}^2 x$
a^x	$f(x) = a^x \Rightarrow f'(x) = a^x(\ln a)$
$\log_a x$	$f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x\ln a}$
$\arcsin x$	$f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$
Standard integrals	$\int a^x\,dx = \frac{1}{\ln a}a^x + C$ $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\left(\frac{x}{a}\right) + C$, $\lvert x\rvert < a$
Integration by parts	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$ or $\int u\,dv = uv - \int v\,du$
Area of region enclosed by a curve & y-axis	$A = \int_a^b \lvert x\rvert\,dy$
Volume of revolution about the x or y-axes	$V = \int_a^b \pi y^2\,dx$ or $V = \int_a^b \pi x^2\,dy$
Euler's method	$y_{n+1} = y_n + h\times f(x_n, y_n)$; $x_{n+1} = x_n + h$ h is a constant (step length)
Integrating factor for $y' + P(x)y = Q(x)$	$e^{\int P(x)dx}$
Maclaurin series	$f(x) = f(0) + x f'(0) + \frac{x^2}{2!}f''(0) + \ldots$
Maclaurin series for special functions	$e^x = 1 + x + \frac{x^2}{2!} + \ldots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$