Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Coefficient of x^3 in $(1-2x)^5$ is −80	B1	Can be seen in an expansion but must be simplified correctly.
	Coefficient of x^2 in $(1-2x)^5$ is 40	B1	
	Coefficient of x^3 in $(1+kx)(1-2x)^5$ is $40k - 80 = 20$	M1	Uses the relevant two terms to form an equation = 20 and solves to find k. Condone x^3 appearing in some terms if recovered.
	$(k =)\dfrac{5}{2}$	A1	
		4	

Question	Answer	Marks	Guidance
2	$(-2p)^2 = (2p+6) \times (p+2)$ or $\dfrac{-2p}{2p+6} = \dfrac{p+2}{-2p}$	**M1**	OE. Using "a, b, c then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p+2$ to form a correct relationship in terms of p only
	$(2p^2 - 10p - 12 = 0)$ $p = 6$	**A1**	
	$a = 18$ and $r = -\frac{2}{3}$	**A1**	
	$(s_\infty) =$ *their* $a \div (1 -$ *their* $r)$ $\left(= 18 \div \dfrac{5}{3}\right)$	**M1**	Correct formula used with their values for a and r, $\lvert r \rvert < 1$ Both a & r from the same value of p.
	$(s_\infty =)10.8$	**A1**	OE. A0 if an extra solution given
			SC B2 for $s_\infty = \dfrac{2p+6}{1 - \dfrac{-2p}{2p+6}}$ *or* $\dfrac{2p+6}{1 - \dfrac{p+2}{-2p}}$ ignore any subsequent algebraic simplification.
		5	

Question	Answer	Marks	Guidance
3	$2x^2 + m(2x+1) - 6x - 4(=0)$	*M1	y eliminated and all terms on one side with correct algebraic steps. Condone ± errors
	Using $b^2 - 4ac$ on $2x^2 + x(2m-6) + m - 4 \ (=0)$	DM1	Any use of discriminant with their *a*, *b* and *c* identified correctly.
	$4m^2 - 32m + 68$ **or** $2m^2 - 16m + 34$ **or** $m^2 - 8m + 17$	A1	
	$(2m-8)^2 + k$ **or** $(m-4)^2 + k$ **or** minimum point $(4, k)$ **or** finds $b^2 - 4ac \ (= -4, -16, -64)$	DM1	OE. Any valid method attempted on their 3-term quadratic
	$(m-4)^2 + 1$ oe + always > 0 → 2 solutions for all values of *m* **or** Minimum point (4,1) + (fn) always > 0 → 2 solutions for all values of *m* **or** $b^2 - 4ac < 0$ + no solutions → 2 solutions for the original equation for all values of *m*	A1	Clear and correct reasoning and conclusion without wrong working.
		5	

Question	Answer	Marks	Guidance
4	S_x *and* S_{x+1}	**M1**	Using two values of n in the given formula
	$a = 5$, $d = 2$	**A1 A1**	
	$a + (n - 1)\,d > 200 \rightarrow 5 + 2(k - 1) > 200$	**M1**	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
	Alternative method for question 4		
	$\frac{n}{2}(\,2a + (n-1)\,d\,) \equiv n^2 + 4n \rightarrow \left(\frac{d}{2} = 1,\ a - \frac{1}{2}d = 4\right)$	**M1**	Equating two correct expressions of S_n and equating coefficients of n and n^2
	$d = 2$, $a = 5$	**A1 A1**	
	$a + (n - 1)\,d > 200 \rightarrow 5 + 2(k - 1) > 200$	**M1**	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
	Alternative method for question 4		
	$sum_k - sum_{k-1} \rightarrow k^2 + 4k - (k-1)^2 - 4(k-1)$	**M1 A1**	Using given formula with consecutive expressions subtracted. Allow k+1 and k.
	$2k + 3 > 200$ or $= 200$	**M1 A1**	Simplifying to a linear equation or inequality
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
		5	

Question	Answer	Marks	Guidance
5(a)	0	**B1**	
		1	
5(b)	$(f^{-1}(x)) = \dfrac{x+2}{4}$, $(g^{-1}(x)) = \dfrac{4-x}{x}$ or $\dfrac{4}{x}-1$	**B1 B1**	OE. Sight of correct inverses.
	$x^2 + 6x - 16$ (= 0)	**B1**	Equating inverses and simplifying.
	$(x+8)$ and $(x-2)$	**M1**	Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula.
	(x =) 2 or −8	**A1**	Do not accept answers obtained with no method shown.
		5	

Question	Answer	Marks	Guidance
6(a)	$\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}+1\right)$	**B1**	Uses "tan$x$ = sin$x$ ÷ cos$x$" throughout
	$\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ or $\left(\frac{1-\sin^2 x}{\cos x\sin x}\right)$	**M1**	Correct algebra leading to two or four terms
	$\left(\frac{\cos^2 x}{\cos x\sin x}\right)$	**A1**	OE. A correct expression which can be cancelled directly to $\frac{\cos x}{\sin x}$ e.g. $\frac{\cos x\left(1-\sin^2 x\right)}{\sin x\left(1-\sin^2 x\right)}$
	$\left(\frac{\cos^2 x}{\cos x\sin x}\right)=\left(\frac{\cos x}{\sin x}\right)=\frac{1}{\tan x}$	**A1**	AG. Must show cancelling. If x is missing throughout their working withhold this mark.
		4	
6(b)	Uses **(a)** → $\frac{1}{\tan x}=2\tan^2 x$ $\tan^3 x=\frac{1}{2}$	**M1**	Reducing to $\tan^3 x = k$.
	$(x =)$ 38.4°	**A1**	AWRT. Ignore extra answers outside the range 0 to 180° but A0 if within.
		2	

Question	Answer	Marks	Guidance
7(a)	$\mathrm{f}'(4)\left(=\frac{5}{2}\right)$	*M1	Substituting 4 into $\mathrm{f}'(x)$
	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}\right)\to\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)=\frac{5}{2}\times 0.12$	DM1	Multiplies *their* $\mathrm{f}'(4)$ by 0.12
	$\left(\frac{\mathrm{d}y}{\mathrm{d}t}=\right)0.3$	A1	OE
		3	
7(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$	B1 B1	B1 for each unsimplified integral.
	Uses (4, 7) leading to $c = (-21)$	M1	Uses (4, 7) to find a c value
	y or $\mathrm{f}(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$	A1	Need to see y or $\mathrm{f}(x)$ = somewhere in *their* solution and 12 and 8
		4	

Question	Answer	Marks	Guidance
8(a)	Use of correct formula for the area of triangle *ABC*	M1	Use of 180–2θ scores M0. Condone 2π–2θ
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	A1	OE
	[Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$	B1 FT	FT for *their* triangle area $-\frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1)
		3	
8(b)	Arc *BD* = $r\theta$ = 6 cm	B1	SOI
	$AC = 2r\cos\theta$ = (2×10cos0.6 = 20cos0.6 = 16.506) or $\sqrt{\left(2r^2-2r^2\cos(\pi-2\theta)\right)}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	*M1	Finding *AC* or $\frac{1}{2}AC$ (= 8.25)
	$DC = 2r\cos\theta - r$ or $\sqrt{\left(2r^2-2r^2\cos(\pi-2\theta)\right)} - r$ (= 6.506)	DM1	Subtracting r from *their AC* or r-$r\cos\theta$ from *their* half *AC* (8.25-1.75)
	(Perimeter = 10 + 6 + 6.506 =) 22.5	A1	AWRT
		4	

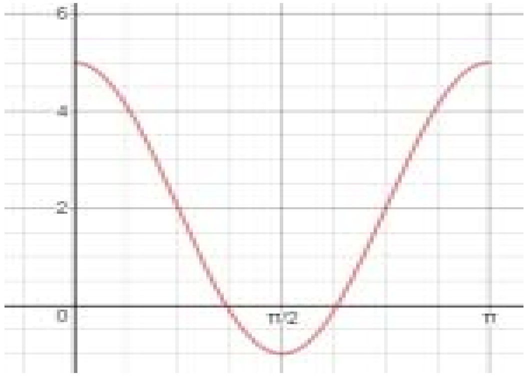
Question	Answer	Marks	Guidance
9(a)	$r = \sqrt{(6^2+3^2)}$ or $r^2 = 45$	**B1**	Sight of r = 6.7 implies B1
	$(x-5)^2+(y-1)^2=r^2$ or $x^2-10x+y^2-2y=r^2-26$	**M1**	Using centre given and *their* radius or r in correct formula
	$(x-5)^2+(y-1)^2=45$ or $x^2-10x+y^2-2y=19$	**A1**	Do not allow $\left(\sqrt{45}\right)^2$ for r^2
		3	
9(b)	*C* has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of *AB, BC* or *AC*.
	Grad of CD $=-2$	**M1**	Calculation of gradient needs to be shown for this M1.
	($\frac{1}{2}\times -2=-1$) then states + perpendicular → hence shown or tangent	**A1**	Clear reasoning needed.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of *AB, BC* or *AC*.
	Gradient of the perpendicular is −2 → Equation of the perpendicular is $y-4=-2(x-11)$	**M1**	Use of $m_1m_2=-1$ with *their* gradient of *AB, BC* or *AC* and correct method for the equation of the perpendicular. Could use *D*(5, 16) instead of *C*(11,4).
	Checks *D*(5, 16) **or** checks gradient of *CD* and then states *D* lies on the line **or** *CD* has gradient −2→ hence shown or tangent	**A1**	Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient.

Question	Answer	Marks	Guidance
9(b)	**Alternative method for question 9(b)**		
	C has coordinates (11, 4) **or** Gradient of *AB, BC* or *AC* = 0.5	**B1**	Only one of *AB, BC* or *AC* needed.
	Equation of the perpendicular is $y-4=-2(x-11)$	**B1**	Finding equation of *CD.*
	$(x-5)^2+(-2x+26-1)^2=45\to(x^2-22x+121=0)$	**M1**	Solving simultaneously with the equation of the circle.
	$(x-11)^2=0$ or $b^2-4ac=0$ → repeated root → hence shown or tangent	**A1**	Must state repeated root.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding $CD=\sqrt{180}$ and $BD=\sqrt{225}$	**B1**	OE. Calculated from the co-ordinates of *B*, *C* & *D* without using *r*.
	Checking (their BD)^2 – (their CD)^2 is the same as (their r)^2	**M1**	
	∴ Pythagoras valid ∴ perpendicular → hence shown or tangent	**A1**	Triangle *ACD* could be used instead.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding vectors $\overrightarrow{AC}$ and $\overrightarrow{CD}$ **or** $\overrightarrow{BC}$ and $\overrightarrow{CD}$ $\left(=\begin{pmatrix}6\\3\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right.$ **or** $\left.\begin{pmatrix}12\\6\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right)$	**B1**	Must be correct pairing.
	Applying the scalar product to one of these pairs of vectors	**M1**	Accept *their* $\overrightarrow{AC}$ and $\overrightarrow{CD}$ or *their* $\overrightarrow{BC}$ and $\overrightarrow{CD}$
	Scalar product = 0 then states ∴ perpendicular → hence shown or tangent	**A1**	
		4	

Question	Answer	Marks	Guidance
9(c)	$E\,(-1, 4)$	**B1 B1**	WWW B1 for each coordinate Note: Equation of DE which is $y = 2x + 6$ may be used to find E
		2	

Question	Answer	Marks	Guidance
10(a)	$(\frac{dy}{dx}) = [8] \times [(3-2x)^{-3}] + [-1]$ $\left(= \frac{8}{(3-2x)^3} - 1\right)$	**B2, 1, 0**	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2} = -3 \times 8 \times (3-2x)^{-4} \times (-2)$ $\left(= \frac{48}{(3-2x)^4}\right)$	**B1 FT**	FT providing *their* bracket is to a negative power
	$\int y\,dx = [(3-2x)^{-1}]\,[2 \div (-1 \times -2)]\,[-\tfrac{1}{2}x^2]\,(+c)$ $\left(= \frac{1}{3-2x} - \frac{1}{2}x^2 + c\right)$	**B1 B1 B1**	Simplification not needed, B1 for each correct element
		6	

Question	Answer	Marks	Guidance
10(b)	$\frac{dy}{dx} = 0 \rightarrow (3-2x)^3 = 8 \rightarrow 3-2x = k \rightarrow x =$	**M1**	Setting *their* 2-term differential to 0 and attempts to solve as far as $x =$
	$\frac{1}{2}$	**A1**	
	Alternative method for question 10(b)		
	$y=0 \rightarrow \frac{2}{(3-2x)^2} - x = 0 \rightarrow (x-2)(2x-1)^2 = 0 \rightarrow x =$	**M1**	Setting y to 0 and attempts to solve a cubic as far as $x =$ (3 factors needed)
	$\frac{1}{2}$	**A1**	
		2	
10(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)} - \frac{\left(\frac{1}{2}\right)^2}{2}\right] - \left[\frac{1}{3-2\times 0} - 0\right]$	**M1**	Using *their* integral, *their* positive x limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	**A1**	
		2	

Question	Answer	Marks	Guidance
11(a)	5, –1	**B1 B1**	Sight of each value
		2	
11(b)		***B1**	Needs to be a curve, not straight lines. One complete cycle starting and finishing at *their* largest value.
		DB1	One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end.
		2	
11(c)(i)	0 solution	**B1**	
		1	
11(c)(ii)	2 solutions	**B1**	
		1	
11(c)(iii)	1 solution	**B1**	
		1	

Question	Answer	Marks	Guidance
11(d)	Stretch by (scale factor) $\frac{1}{2}$, parallel to x-axis or in x direction (or horizontally)	**B1**	
	Translation of $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$	**B1**	Accept translation/shift Accept translation 4 units in positive y-direction.
		2	
11(e)	Translation of $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$	**B1**	Accept translation/shift Accept translation $-\frac{\pi}{2}$ units in x-direction.
	Stretch by (scale factor) 2 parallel to y-axis (or vertically).	**B1**	
		2	

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