

Mathematics: analysis and approaches
Standard level
Paper 1

Save My Exams Practice Paper

1 hour 30 minutes

Instructions to candidates

- You are not permitted access to any calculator for this paper.
- Section A: answer all questions.
- Section B: answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

1. [Maximum mark: 5]

Let A and B be events such that $P(A) = 0.3$, $P(B) = 0.75$ and $P(A \cup B) = 0.9$.
Find $P(B | A)$.

2. [Maximum mark: 5]

Given that $\frac{dy}{dx} = 3x^2 \cos\left(3x^3 + \frac{\pi}{2}\right)$ and that the graph of y passes through the point $(0, -1)$, find an expression for y in terms of x .

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3. [Maximum mark: 5]

The functions f and g are defined such that $f(x) = 6x + 7$ and $g(x) = \frac{x-5}{3}$.

(a) Show that $(f \circ g)(x) = 2x - 3$. [2]

(b) Given that $(f \circ g)^{-1}(a) = 6$, find the value of a . [3]

4. [Maximum mark: 5]

(a) (i) Expand $(2k - 1)^3$.

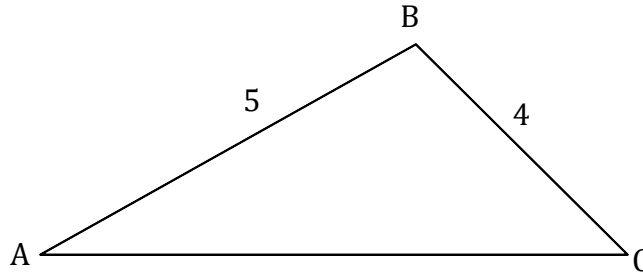
(ii) Hence, or otherwise, show that $(2k - 1)^3 - (2k - 1) = 8k^3 - 12k^2 + 4k$. [2]

(b) Thus prove, given $k > 1$, $k \in \mathbb{N}$, that the difference between an odd natural number greater than 1 and its cube is always even. [3]

5. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 5$ and $BC = 4$.

diagram not to scale



- (a) (i) Given that $\sin \hat{B} = \frac{3}{5}$, find the possible values of $\cos \hat{B}$.
- (ii) Given that \hat{B} is obtuse, find the precise value of $\cos \hat{B}$. [3]
- (b) Find the length of AC. [2]

6. [Maximum mark: 8]

(a) Show that $\log_4(\cos 2x + 13) = \log_2 \sqrt{\cos 2x + 13}$. [3]

(b) Hence or otherwise solve $\log_2(3\sqrt{2} \cos x) = \log_4(\cos 2x + 13)$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$. [5]

Section B

7. [Maximum mark: 16]

Let $f(x) = \frac{1}{3}x^3 - 2x^2 - 21x - 24$.

(a) Find $f'(x)$. [2]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(b) Find the value of a and the value of b . [3]

(c) (i) Find $f''(x)$.

(ii) Hence show that the graph of f has a local maximum point at $x = a$. [2]

(d) (i) Sketch the graph of $y = f'(x)$.

(ii) Hence, use your answer to part (d)(i) to explain why the graph of f has a local minimum point at $x = b$. [4]

The tangent to the graph of f at $x = a$ and the normal to the graph of f at $x = b$ intersect at the point (p, q) .

(e) Find the value of p and the value of q . [5]

8. [Maximum mark: 16]

Let $f(x) = \frac{\ln px}{qx}$ where $x > 0$, $p, q \in \mathbb{R}^+$.

(a) Show that $f'(x) = \frac{1 - \ln px}{qx^2}$. [3]

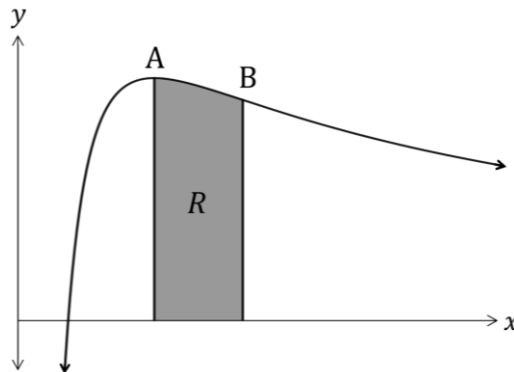
The graph of f has exactly one maximum point A.

(b) Find the x -coordinate of A. [3]

The second derivative of f is given by $f''(x) = \frac{2 \ln px - 3}{qx^3}$. The graph of f has exactly one point of inflexion B.

(c) Show that the x -coordinate of B is $\frac{e^{\frac{3}{2}}}{p}$. [3]

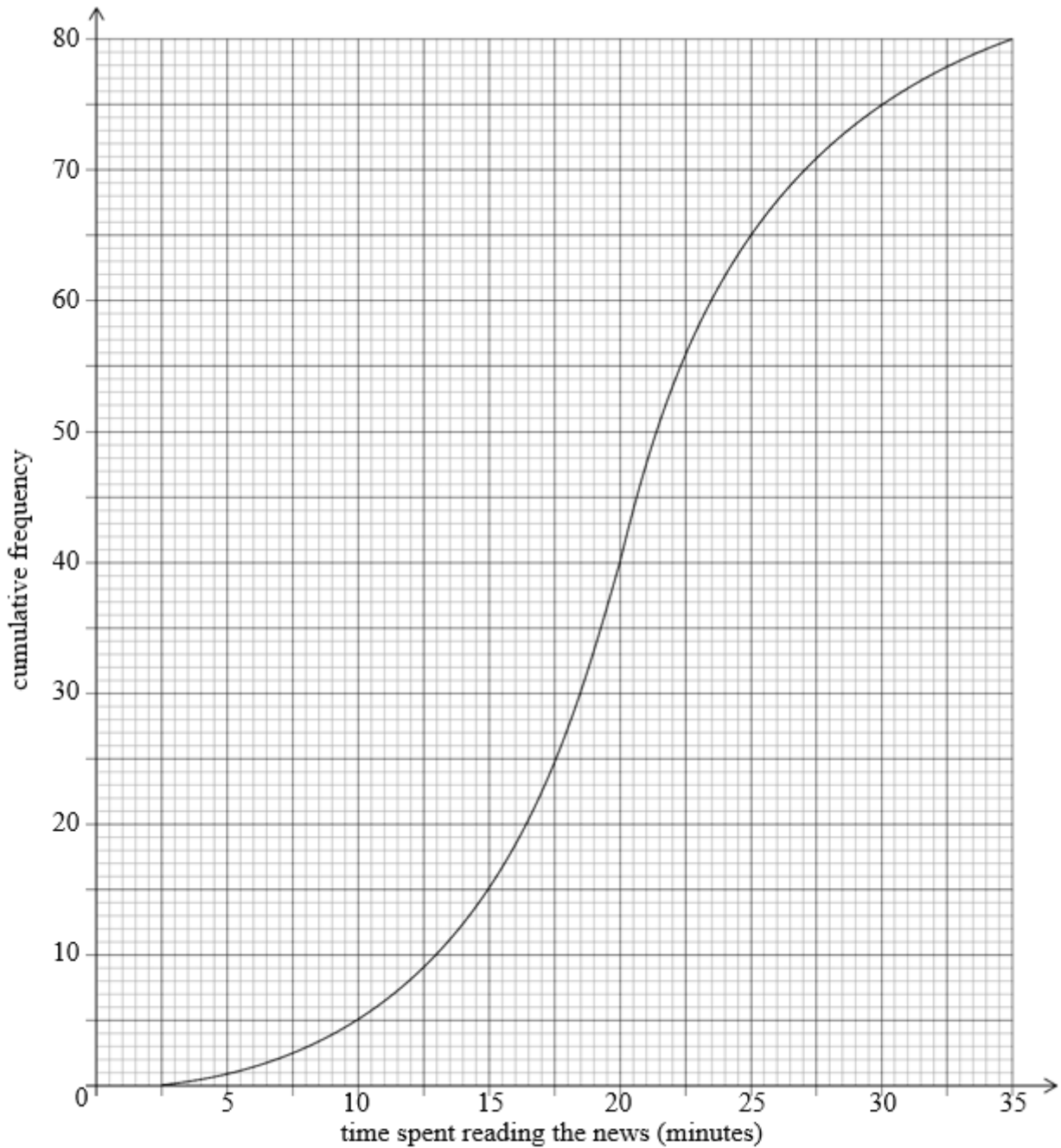
The region R is enclosed by the graph of f , the x -axis, and the vertical lines through the maximum point A and the point of inflexion B.



(d) Calculate the area of R in terms of q and show that the value of the area is independent of p . [7]

9. [Maximum mark: 15]

A school surveyed 80 of its final year students to find out how much time they spent reading the news on a given day. The results of the survey are shown in the following cumulative frequency diagram.



(This question continues on the following page)

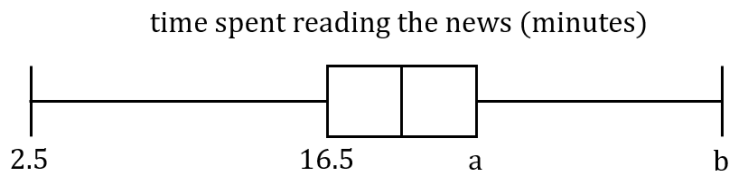
(Question 9 continued)

- (a) Find the median number of minutes spent reading the news. [2]
- (b) Find the number of students whose reading time is within 2.5 minutes of the median. [3]

Only 15% of students spent more than k minutes reading.

- (c) Find the value of k . [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.



- (d) Write down the value of b . [1]
- (e) (i) Find the value of a .
- (ii) Hence, find the interquartile range. [4]
- (f) Determine whether someone who spends 30 minutes reading the news would be an outlier. [2]