IB Maths: Applications & Interpretation SL & HL

Formula sheet for use during the IB Maths course & examinations

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Prior Learning SL & HL

Area of a parallelogram	$A = bh$	b is the base, h is the height
Area of a triangle	$A = \frac{1}{2}(bh)$	b is the base, h is the height
Area of a trapezium	$A = \frac{1}{2}(a+b)h$	a and b are the parallel sides, h is the height
Area of a circle	$A = \pi r^2$	r is the radius
Circumference of a circle	$C = 2\pi r$	r is the radius
Volume of a cuboid	$V = lwh$	l is the length, w is the width, h is the height
Volume of a cylinder	$V = \pi r^2 h$	r is the radius, h is the height
Volume of a prism	$V = Ah$	A is the area of cross-section, h is the height
Area of the curved surface of a cylinder	$A = 2\pi rh$	r is the radius, h is the height
Distance between two points (x_1, y_1) and (x_2, y_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1) and (x_2, y_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	

Prior learning – HL only

Solutions of a quadratic equation	The solutions of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$

Topic 1: Number & Algebra – SL & HL

The n th term of an arithmetic sequence	$u_n = u_1 + (n-1)d$	
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d)$; $S_n = \frac{n}{2}(u_1 + u_n)$	
The n th term of a geometric sequence	$u_n = u_1 r^{n-1}$	
The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$	
Compound interest	$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$	FV is the future value, PV is the present value, n is the number of years, k is the number of compounding periods per year, r% is the nominal annual rate of interest
Exponents & logarithms	$a^x = b \Leftrightarrow x = \log_a b$	$a > 0, b > 0, a \neq 1$
Percentage error	$E = \left\lvert \frac{v_A - v_E}{v_E} \right\rvert \times 100\%$	v_A is the exact value and v_A is the approximate value of v

Topic 1: Number & Algebra – HL only

Laws of logarithms	$\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ for $a, x, y > 0$
The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1 - r}$, $\lvert r \rvert < 1$
Complex numbers	$z = a + bi$
Discriminant	$\Delta = b^2 - 4ac$
Modulus-argument (polar) & exponential (Euler) form	$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\mathrm{cis}\,\theta$
Determinant of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det A = \lvert A \rvert = ad - bc$
Inverse of a 2×2 matrix	$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, $ad \neq bc$
Power formula for a matrix	$M^n = PD^nP^{-1}$	P is the matrix of eigenvectors and D is the diagonal matrix of eigenvalues

Topic 2: Functions – SL & HL

Equations of a straight line	$y = mx + c$; $ax + by + d = 0$; $y - y_1 = m(x - x_1)$
Gradient formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Axis of symmetry of the graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$

Topic 2: Functions – HL only

Logistic function	$f(x) = \frac{L}{1 + Ce^{-kx}}$, $L, k, C > 0$

Topic 3: Geometry & Trigonometry – SL & HL

Distance between two points (x_1, y_1, z_1) & (x_2, y_2, z_2)	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$	
Coordinates of the midpoint of a line segment with endpoints (x_1, y_1, z_1) & (x_2, y_2, z_2)	$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$	
Volume of a right-pyramid	$V = \frac{1}{3}Ah$	A is the area of the base, h is the height
Volume of a right cone	$V = \frac{1}{3}\pi r^2 h$	r is the radius, h is the height
Area of the curved surface of a cone	$A = \pi rl$	r is the radius, l is the slant height
Volume of a sphere	$V = \frac{4}{3}\pi r^3$	r is the radius
Surface area of a sphere	$A = 4\pi r^2$	r is the radius
Sine rule	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	
Cosine rule	$c^2 = a^2 + b^2 - 2ab\cos C$; $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$	
Area of a triangle	$A = \frac{1}{2}ab\sin C$	
Length of an arc	$l = \frac{\theta}{360} \times 2\pi r$	θ is the angle measured in degrees, r is the radius
Area of a sector	$A = \frac{\theta}{360} \times \pi r^2$,	θ is the angle measured in degrees, r is the radius

Topic 3: Geometry & Trigonometry – HL only

Length of an arc	$l = r\theta$	r is the radius, θ is the angle measured in radians
Area of a sector	$A = \frac{1}{2}r^2\theta$	r is the radius θ is the angle measured in radians
Identities	$\cos^2\theta + \sin^2\theta = 1$	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Transformation matrices	$\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$,	reflection in the line $y = (\tan\theta)x$
	$\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$,	horizontal stretch / stretch parallel to x-axis with a scale factor of k
	$\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}$,	vertical stretch/ stretch parallel to y-axis with a scale factor of k
	$\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$,	enlargement, with a scale factor of k, centre $(0, 0)$
	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$,	anticlockwise/counter-clockwise rotation of angle θ about the origin ($\theta > 0$)
	$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$,	clockwise rotation of angle θ about the origin ($\theta > 0$)
Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$	$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$
Vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$	
Parametric form of the equation of a line	$x = x_0 + \lambda l$, $y = y_0 + \lambda m$, $z = z_0 + \lambda n$	
Scalar product	$\boldsymbol{v} \cdot \boldsymbol{w} = v_1w_1 + v_2w_2 + v_3w_3$	$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$
	$\boldsymbol{v} \cdot \boldsymbol{w} = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \cos\theta$	θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Angle between two vectors	$\cos\theta = \frac{v_1w_1 + v_2w_2 + v_3w_3}{\lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert}$	
Vector product	$\boldsymbol{v} \times \boldsymbol{w} = \begin{pmatrix} v_2w_3 - v_3w_2 \\ v_3w_1 - v_1w_3 \\ v_1w_2 - v_2w_1 \end{pmatrix}$,	$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$, $\boldsymbol{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$
	$\lvert \boldsymbol{v} \times \boldsymbol{w} \rvert = \lvert \boldsymbol{v} \rvert \lvert \boldsymbol{w} \rvert \sin\theta$,	θ is the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
Area of a parallelogram	$A = \lvert \boldsymbol{v} \times \boldsymbol{w} \rvert$	$\boldsymbol{v}$ and $\boldsymbol{w}$ form two adjacent sides of a parallelogram

Topic 4: Statistics & Probability – HL only

Linear transformation of a single random variable	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{Var}(aX + b) = a^2\,\mathrm{Var}(X)$
Linear combinations of n independent random variables, $X_1, X_2, ..., X_n$	$\mathrm{E}(a_1X_1 \pm a_2X_2 \pm ... \pm a_nX_n) = a_1\mathrm{E}(X_1) \pm a_2\mathrm{E}(X_2) \pm ... \pm a_n\mathrm{E}(X_n)$ $\mathrm{Var}(a_1X_1 \pm a_2X_2 \pm ... \pm a_nX_n)$ $= a_1^2\,\mathrm{Var}(X_1) + a_2^2\,\mathrm{Var}(X_2) + ... + a_n^2\,\mathrm{Var}(X_n)$
Sample statistics	
Unbiased estimate of population variance s_{n-1}^2	$s_{n-1}^2 = \frac{n}{n-1}s_n^2$
Poisson distribution $X \sim \mathrm{Po}(m)$	
Mean	$\mathrm{E}(X) = m$
Variance	$\mathrm{Var}(X) = m$
Transition matrices	$T^n\boldsymbol{s}_0 = \boldsymbol{s}_n$	$\boldsymbol{s}_0$ is the initial state

Topic 4: Statistics & Probability – SL & HL

Interquartile range	$\mathrm{IQR} = Q_3 - Q_1$
Mean, $\bar{x}$, of a set of data	$\bar{x} = \frac{\sum_{i=1}^{k} f_i x_i}{n}$ $\quad n = \sum_{i=1}^{k} f_i$
Probability of an event A	$\mathrm{P}(A) = \frac{n(A)}{n(U)}$
Complementary events	$\mathrm{P}(A) + \mathrm{P}(A') = 1$
Combined events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
Mutually exclusive events	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
Conditional probability	$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
Independent events	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$
Expected value of a discrete random variable X	$\mathrm{E}(X) = \sum x\,\mathrm{P}(X = x)$
Binomial distribution $X \sim \mathrm{B}(n, p)$	
Mean	$\mathrm{E}(X) = np$
Variance	$\mathrm{Var}(X) = np(1 - p)$

Topic 5: Calculus – SL & HL

Derivative of x^n	$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$
Integral of x^n	$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C$, $\quad n \neq -1$
Area of region enclosed by a curve $y = f(x)$ & the x-axis, where $f(x) > 0$	$A = \int_a^b y\,\mathrm{d}x$
The trapezium rule	$\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\left((y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\right)$, $\quad h = \frac{b-a}{n}$

Topic 5: Calculus – HL only

Derivative of $\sin x$	$f(x) = \sin x \Rightarrow f'(x) = \cos x$
Derivative of $\cos x$	$f(x) = \cos x \Rightarrow f'(x) = -\sin x$
Derivative of $\tan x$	$f(x) = \tan x \Rightarrow f'(x) = \frac{1}{\cos^2 x}$
Derivative of e^x	$f(x) = e^x \Rightarrow f'(x) = e^x$
Derivative of $\ln x$	$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$
Chain rule	$y = g(u)$ $\quad u = f(x) \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$
Product rule	$y = uv \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$
Quotient rule	$y = \frac{u}{v} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$
Standard integrals	$\int \frac{1}{x}\,\mathrm{d}x = \ln\lvert x \rvert + C$
	$\int \sin x\,\mathrm{d}x = -\cos x + C$
	$\int \cos x\,\mathrm{d}x = \sin x + C$
	$\int \frac{1}{\cos^2 x} = \tan x + C$
	$\int e^x\,\mathrm{d}x = e^x + C$
Area of region enclosed by a curve and x or y-axes	$A = \int_a^b \lvert y \rvert\,\mathrm{d}x$ or $A = \int_a^b \lvert x \rvert\,\mathrm{d}y$
Volume of revolution about x or y-axes	$V = \int_a^b \pi y^2\,\mathrm{d}x$ or $V = \int_a^b \pi x^2\,\mathrm{d}y$
Acceleration	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2s}{\mathrm{d}t^2} = v\frac{\mathrm{d}v}{\mathrm{d}s}$
Distance travelled from t_1 to t_2	$\text{distance} = \int_{t_1}^{t_2} \lvert v(t) \rvert\,\mathrm{d}t$
Displacement from t_1 to t_2	$\text{displacement} = \int_{t_1}^{t_2} v(t)\,\mathrm{d}t$
Euler's method	$y_{n+1} = y_n + h \times f(x_n, y_n)$; $x_{n+1} = x_n + h$	h is a constant (step length)
Euler's method for coupled systems	$x_{n+1} = x_n + h \times f_1(x_n, y_n, t_n)$ $y_{n+1} = y_n + h \times f_2(x_n, y_n, t_n)$ $t_{n+1} = t_n + h$	h is a constant (step length)
Exact solution for coupled linear differential equations	$\boldsymbol{x} = Ae^{\lambda_1 t}\boldsymbol{p}_1 + Be^{\lambda_2 t}\boldsymbol{p}_2$	

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Level	Paper	Length	Marks
SL	1	90 mins	80
SL	2	90 mins	80

Level	Paper	Length	Marks
HL	1	2 hours	110
HL	2	2 hours	110
HL	3	1 hour	55