Sequences
Difficulty: Hard

Question Paper 2

Level	IGCSE
Subject	Maths (0580/0980)
Exam Board	CIE
Topic	Sequences
Paper	Paper 4
Difficulty	Hard
Booklet	Question Paper 2

Time allowed: **94 minutes**

Score: **/82**

Percentage: **/100**

Grade Boundaries:

CIE IGCSE Maths (0580)

A*	A	B	C	D
>83%	67%	51%	41%	31%

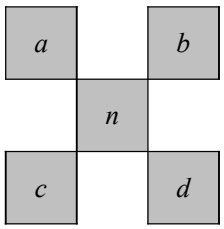
CIE IGCSE Maths (0980)

9	8	7	6	5	4
>95%	87%	80%	69%	58%	46%

Consecutive integers are set out in rows in a grid.

(a) This grid has 5 columns.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35



The shape drawn encloses five numbers 7, 9, 13, 17 and 19. This is the $n = 13$ shape.

In this shape, $a = 7$, $b = 9$, $c = 17$ and $d = 19$.

(i) Calculate $bc - ad$ for the $n = 13$ shape. [1]

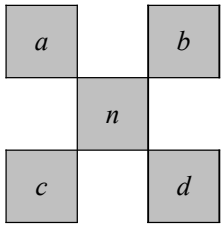
(ii) For the 5 column grid, $a = n - 6$.

Write down b, c and d in terms of n for this grid. [2]

(iii) Write down $bc - ad$ in terms of n.
Show clearly that it simplifies to 20. [2]

(b) This grid has 6 columns. The shape is drawn for $n = 10$.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

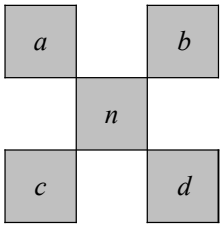


(i) Calculate the value of $bc - ad$ for $n = 10$. [1]

(ii) Without simplifying, write down $bc - ad$ in terms of n for this grid. [2]

(c) This grid has 7 columns.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35

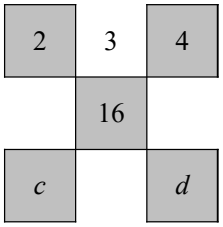


Show clearly that $bc - ad = 28$ for $n = 17$.

Answer(c) [1]

(d) Write down the value of $bc - ad$ when there are t columns in the grid. [1]

(e) Find the values of c, d and $bc - ad$ for this shape.



[2]

(a) Complete the table for the 6 th term and the nth term in each sequence.

	Sequence	6 th term		n th term
A	11, 9, 7, 5, 3			
B	1, 4, 9, 16, 25			
C	2, 6, 12, 20, 30			
D	3, 9, 27, 81, 243			
E	1, 3, 15, 61, 213			

[12]

(b) Find the value of the 100 th term in

(i) Sequence A, [1]

(ii) Sequence C. [1]

(c) Find the value of n in Sequence D when the n th term is equal to 6561. [1]

(d) Find the value of the 10 th term in Sequence E. [1]

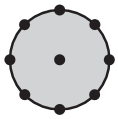


Diagram 1

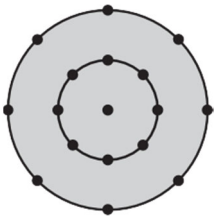


Diagram 2

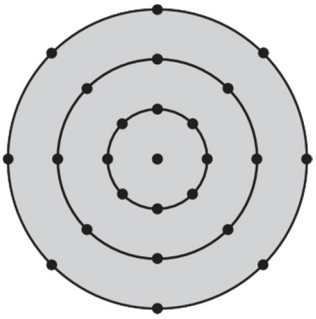


Diagram 3

The diagrams show a sequence of dots and circles.
Each diagram has one dot at the centre and 8 dots on each circle.
The radius of the first circle is 1 unit.
The radius of each new circle is 1 unit greater than the radius of the previous circle.

(a) Complete the table for diagrams 4 and 5.

Diagram	1	2	3	4	5
Number of dots	9	17	25		
Area of the largest circle	π	4π	9π		
Total length of the circumferences of the circles	2π	6π	12π		

[4]

(b) (i) Write down, in terms of n, the number of dots in diagram n. [2]

(ii) Find n, when the number of dots in diagram n is 1097. [2]

(c) Write down, in terms of n and π, the area of the largest circle in

(i) diagram n, [1]

(ii) diagram $3n$. [1]

(d) Find, in terms of n and π, the total length of the circumferences of the circles in diagram n. [2]

The first and the nth terms of sequences A, B and C are shown in the table below.

(a) Complete the table for each sequence. [5]

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence A	1					n^3
Sequence B	4					$4n$
Sequence C	4					$(n+1)^2$

(b) Find

(i) the 8th term of sequence A, [1]

(ii) the 12th term of sequence C. [1]

(c) (i) Which term in sequence A is equal to 15625? [1]

(ii) Which term in sequence C is equal to 10000? [1]

(d) The first four terms of sequences D and E are shown in the table below.

Use the results from **part (a)** to find the 5th and the nth terms of the sequences D and E. [4]

	1st term	2nd term	3rd term	4th term	5th term	nth term
Sequence D	5	16	39	80		
Sequence E	0	1	4	9		

(a) (i) Work out the first 3 terms of the sequence whose nth term is $n(n + 2)$. [2]

(ii) Which term in this sequence is equal to 168? [3]

(b) Find a formula for the nth term of the following sequences.

(i) 5 8 11 14 17 …… [2]

(ii) 1 2 4 8 16 …… [2]

(c)

Diagram 1 Diagram 2 Diagram 3

A sequence of diagrams is formed by drawing equilateral triangles each of side one centimetre.
Diagram 1 has 3 one centimetre lines.
Diagram 2 has 9 one centimetre lines.

The formula for the **total** number of one centimetre lines needed to draw all of the first n **diagrams** is

$$an^3 + bn^2 + n.$$ [6]

Find the values of a and b.

(a) (i) The first three positive integers 1, 2 and 3 have a sum of 6.

Write down the sum of the first 4 positive integers. [1]

(ii) The formula for the sum of the first n integers is $\frac{n(n+1)}{2}$.

Show the formula is correct when $n = 3$. [1]

(iii) Find the sum of the first 120 positive integers. [1]

(iv) Find the sum of the integers

$121 + 122 + 123 + 124 +$ ..... $+ 199 + 200.$ [2]

(v) Find the sum of the even numbers

$2 + 4 + 6 +$ ..... $+ 800.$ [2]

(b) (i) Complete the following statements about the sums of cubes and the sums of integers. [2]

$1^3 = 1$	$1 = 1$
$1^3 + 2^3 = 9$	$1 + 2 = 3$
$1^3 + 2^3 + 3^3 =$	$1 + 2 + 3 =$ .....
$1^3 + 2^3 + 3^3 + 4^3 =$	$1 + 2 + 3 + 4 =$ .....

(ii) The sum of the first 14 integers is 105.

Find the sum of the first 14 cubes. [1]

(iii) Use the formula in part(a)(ii) to write down a formula for the sum of the first n cubes. [1]

(iv) Find the sum of the first 60 cubes. [1]

(v) Find n when the sum of the first n cubes is 278784. [2]