## Graphs

## Difficulty: Hard

## Question Paper 4

| Level | IGCSE |
| :--- | :--- |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Graphs |
| Paper | Paper 4 |
| Difficulty | Hard |
| Booklet | Question Paper 4 |

Time allowed: 101 minutes
Score:

/88
Percentage: /100

## Grade Boundaries:

CIE IGCSE Maths (0580)

| A* | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| $>83 \%$ | $67 \%$ | $51 \%$ | $41 \%$ | $31 \%$ |

CIE IGCSE Maths (0980)

| 9 | 8 | 7 | 6 | 5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $>95 \%$ | $87 \%$ | $80 \%$ | $69 \%$ | $58 \%$ | $46 \%$ |

Answer the whole of this question on one sheet of graph paper.

$$
\mathrm{f}(x)=1-\frac{1}{x^{2}}, x \neq 0
$$

(a)

| $x$ | -3 | -2 | -1 | -0.5 | -0.4 | -0.3 | 0.3 | 0.4 | 0.5 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | 0.75 | 0 | -3 | -5.25 | $q$ | $q$ | -5.25 | -3 | 0 | 0.75 | $p$ |

Find the values of $p$ and $q$.
(b) (i) Draw an $x$-axis for $-3 \leqslant x \leqslant 3$ using 2 cm to represent 1 unit and a $y$-axis for $-11 \leqslant y \leqslant 2$ using 1 cm to represent 1 unit.
(ii) Draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.3$ and for $0.3 \leqslant x \leqslant 3$.
(c) Write down an integer $k$ such that $\mathrm{f}(x)=k$ has no solutions.
(d) On the same grid, draw the graph of $y=2 x-5$ for $-3 \leqslant x \leqslant 3$.
(e) (i) Use your graphs to find solutions of the equation

$$
\begin{equation*}
1-\frac{1}{x^{2}}=2 x-5 \tag{3}
\end{equation*}
$$

(ii) Rearrange $1-\frac{1}{\mathrm{x}^{2}}=2 \mathrm{x}-5$ into the form $a x^{3}+b x^{2}+c=0$, where $a, b$ and $c$ are integers.
(f) (i) Draw a tangent to the graph of $y=\mathrm{f}(x)$ which is parallel to the line $y=2 x-5$.
(ii) Write down the equation of this tangent.

Answer the whole of this question on a sheet of graph paper.
(a)

$$
\mathrm{f}(x)=\frac{12}{\mathrm{x}+1}
$$

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | 6 | 4 | 3 | 2.4 | 2 | 1.71 | $q$ | 1.33 | $r$ | 1.09 | 1 |

(i) Calculate the values of $p, q$ and $r$.
(ii) Draw the graph of $y=\mathrm{f}(x)$ for $0 \leqslant x \leqslant 11$.

Use a scale of 1 cm to 1 unit on each axis.
(iii) By drawing a suitable line, find an estimate of the gradient of the graph at the point $(3,3)$.
(b) On the same grid draw the graph of $y=8-x$ for $0 \leqslant x \leqslant 8$.
(c) (i) Show that the equation $\mathrm{f}(x)=8-x$ simplifies to $x^{2}-7 \mathrm{x}+4=0$.
(ii) Use your graph to solve this equation, giving your answers correct to 1 decimal place.

Answer all of this question on a sheet of graph paper.
(a) $f(x)=x^{2}-x-3$.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | 3 | -I | -3 | $q$ | -I | 3 | $r$ |

(i) Find the values of $p, q$ and $r$.
(ii) Draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant 4$.

Use a scale of 1 cm to represent 1 unit on each axis.
(iii) By drawing a suitable line, estimate the gradient of the graph at the point where $x=-1$.
(b) $g(x)=6-\frac{x^{3}}{3}$.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}(x)$ | 8.67 | $u$ | $v$ | 5.67 | 3.33 | -3 |

(i) Find the values of $u$ and $v$.
(ii) On the same grid as part (a) (ii) draw the graph of $y=\mathrm{g}(x)$ for $-2 \leqslant x \leqslant 3$.
(c) (i) Show that the equation $\mathrm{f}(x)=\mathrm{g}(x)$ simplifies to $x^{3}+3 x^{2}-3 x-27=0$.
(ii) Use your graph to write down a solution of the equation $x^{3}+3 x^{2}-3 x-27=0$.

Answer the whole of this question on a sheet of graph paper.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(t)$ | 0 | 25 | 37.5 | 43.8 | 46.9 | 48.4 | 49.2 | 49.6 |

(a) Using a scale of 2 cm to represent 1 unit on the horizontal $t$-axis and 2 cm to represent 10 units on the $y$-axis, draw axes for $0 \leq t \leq 7$ and $0 \leq y \leq 60$.
Draw the graph of the curve $y=\mathrm{f}(t)$ using the table of values above.
(b) $\mathrm{f}(t)=50\left(1-2^{-t}\right)$.
(i) Calculate the value of $f(8)$ and the value of $f(9)$.
(ii) Estimate the value of $\mathrm{f}(t)$ when $t$ is large.
(c) (i) Draw the tangent to $y=\mathrm{f}(t)$ at $t=2$ and use it to calculate an estimate of the gradient of the curve at this point.
(ii) The function $\mathrm{f}(t)$ represents the speed of a particle at time $t$.

Write down what quantity the gradient gives.
(d) (i) On the same grid, draw $y=\mathrm{g}(t)$ where $\mathrm{g}(t)=6 t+10$, for $0 \leq t \leq 7$.
(ii) Write down the range of values for $t$ where $\mathrm{f}(t)>\mathrm{g}(t)$.
(iii) The function $\mathrm{g}(t)$ represents the speed of a second particle at time $t$.

State whether the first or second particle travels the greater distance for $0 \leq t \leq 7$.
You must give a reason for your answer.

## Answer the whole of this question on a sheet of graph paper.

| $x$ | 04 | 03 | 02 | 01 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | 08 | 4.5 | 8 | 5.5 | 0 | 05.5 | 08 | 04.5 | 8 |

(a) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 4 units on the $y$-axis, draw axes for $-4 \leq x \leq 4$ and $-8 \leq y \leq 8$.
Draw the curve $y \# \mathrm{f}(x)$ using the table of values given above.
(b) Use your graph to solve the equation $\mathrm{f}(x)=0$.
(c) On the same grid, draw $y=\mathrm{g}(x)$ for $-4 \leq x \leq 4$, where $\mathrm{g}(x)=x+1$.
(d) Write down the value of
(i) $\mathrm{g}(1)$,
(ii) $\mathrm{fg}(1)$,
(iii) $\mathrm{g}^{01}(4)$,
(iv) the positive solution of $\mathrm{f}(x)=\mathrm{g}(x)$.
(e) Draw the tangent to $y=\mathrm{f}(x)$ at $x=3$. Use it to calculate an estimate of the gradient of the curve at this point.

