## Graphs

## Difficulty: Hard

## Question Paper 3

| Level | IGCSE |
| :--- | :--- |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Graphs |
| Paper | Paper 4 |
| Difficulty | Hard |
| Booklet | Question Paper 3 |

Time allowed: $\quad 132$ minutes
Score: /115

Percentage: /100

## Grade Boundaries:

CIE IGCSE Maths (0580)

| A* | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| $>83 \%$ | $67 \%$ | $51 \%$ | $41 \%$ | $31 \%$ |

CIE IGCSE Maths (0980)

| 9 | 8 | 7 | 6 | 5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $>95 \%$ | $87 \%$ | $80 \%$ | $69 \%$ | $58 \%$ | $46 \%$ |

(a) Complete the table of values for the function $y=x^{2}-\underset{\mathrm{x}}{\frac{3}{x}}, x \neq 0$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.25 | 0.25 | 0.5 | 1 | 2 | 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 10 | 5.5 |  | 6.3 | 12.1 |  | -11.9 |  |  | 2.5 | 8 |

(b) Draw the graph of $y=x^{2}-\frac{3}{x}$ for $-3 \leqslant x \leqslant-0.25$ and $0.25 \leqslant x \leqslant 3$.

(c) Use your graph to solve $x^{2}-\frac{3}{x}=7$.
(d) Draw the tangent to the curve where $x=-2$.

Use the tangent to calculate an estimate of the gradient of the curve where $x=-2$.
(a) Complete the table of values for the function $\mathrm{f}(x)$, where $\mathrm{f}(x)=x^{2}+\frac{1}{x^{2}}, x \neq 0$.

| $x$ | -3 | -2.5 | -2 | -1.5 | -1 | -0.5 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ |  | 6.41 |  | 2.69 |  | 4.25 | 4.25 |  | 2.69 |  | 6.41 |  |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 3$.

(c) (i) Write down the equation of the line of symmetry of the graph.
(ii) Draw the tangent to the graph of $y=\mathrm{f}(x)$ where $x=-1.5$.

Use the tangent to estimate the gradient of the graph of $y=\mathrm{f}(x)$ where $x=-1.5$.
(iii) Use your graph to solve the equation $x^{2}+\frac{1}{x^{2}}=3$.
(iv) Draw a suitable line on the grid and use your graphs to solve the equation $x^{2}+\frac{1}{x^{2}}=2 x$.
[3]
(a) Complete the table for the function $\mathrm{f}(x)=\frac{2}{x}-x^{2}$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.2 | 0.2 | 0.5 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | -9.7 | -5 |  |  | -10.0 | 10.0 | 3.75 | 1 |  | -8.3 |

(b) On the grid draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.2$ and $0.2 \leqslant x \leqslant 3$.

(c) Use your graph to
(i) solve $\mathrm{f}(x)=2$,
(ii) find a value for $k$ so that $\mathrm{f}(x)=k$ has 3 solutions.
(d) Draw a suitable line on the grid and use your graphs to solve the equation $\frac{2}{\mathrm{x}}-\mathrm{x}^{2}=5 x$.
(e) Draw the tangent to the graph of $y=\mathrm{f}(x)$ at the point where $x=-2$.

Use it to calculate an estimate of the gradient of $y=\mathrm{f}(x)$ when $x=-2$.


The graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$ are shown above.
(a) Find the value of
(i) $f(-2)$,
(ii) $\mathrm{g}(0)$.
(b) Use the graphs to solve
(i) the equation $\mathrm{f}(x)=20$,
(ii) the equation $\mathrm{f}(x)=\mathrm{g}(x)$,
(iii) the inequality $\mathrm{f}(x)<\mathrm{g}(x)$.
(c) Use the points $A$ and $B$ to find the gradient of $y=\mathrm{g}(x) \quad$ as an exact fraction.
(d) On the grid, draw the graph of $y=\mathrm{g}(x)-10$.
(e) (i) Draw the tangent to the graph of $y=\mathrm{f}(x)$ at ( $-3,-27$ ).
(ii) Write down the equation of thistangent.
(f) A region, $R$, contains points whose co-ordinates satisfy the inequalities

$$
-3 \leqslant x \leqslant-2, \quad y \leqslant 40 \quad \text { and } \quad y \geqslant \mathrm{~g}(x)
$$

On the grid, draw suitable lines and label this region $R$.
(a) The table shows some values for the equation $y=\frac{x}{2}-\frac{2}{x}$ for $-4 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 4$.

| $x$ | -4 | -3 | -2 | -1.5 | -1 | -0.5 | 0.5 | 1 | 1.5 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -1.5 | -0.83 | 0 | 0.58 |  |  | -3.75 |  | -0.58 | 0 | 0.83 | 1.5 |

(i) Write the missing values of $y$ in the empty spaces.
[3]
(ii) On the grid, draw the graph of $y=\frac{x}{2}-\frac{2}{x}$ for $-4 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 4$.

[5]
(b) Use your graph to solve the equation $\frac{x}{2}-\frac{2}{x}=1$.
(c) (i) By drawing a tangent, work out the gradient of the graph where $x=2$.
(ii) Write down the gradient of the graph where $x=-2$.
(d) (i) On the grid, draw the line $y=-x$ for $-4 \leqslant x \leqslant 4$.
(ii) Use your graphs to solve the equation $\frac{\mathrm{x}}{2}-\frac{2}{\mathrm{x}}=-\mathrm{x}$.
(e) Write down the equation of a straight line which passes through the origin and does not intersect the graph of $y=\frac{x}{2}-\frac{2}{x}$.

Answer the whole of this question on a sheet of graph paper.

The table shows some of the values of the function $\mathrm{f}(x)=x^{2}-\frac{1}{x}, \quad x \neq 0$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.2 | 0.2 | 0.5 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 9.3 | 4.5 | 2.0 | 2.3 | $p$ | -5.0 | -1.8 | $q$ | 3.5 | $r$ |

(a) Find the values of $p, q$ and $r$, correct to 1 decimal place.
(b) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 1 cm to represent 1 unit on the $y$-axis, draw an $x$-axis for $-3 \leqslant x \leqslant 3$ and a $y$-axis for $-6 \leqslant y \leqslant 10$.

Draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.2$ and $0.2 \leqslant x \leqslant 3$.
(c) (i) By drawing a suitable straight line, find the three values of $x$ where $\mathrm{f}(x)=-3 x$.
(ii) $x^{2}-\frac{1}{x}=-3 x$ can be written as $x^{3}+a x^{2}+b=0$. Find the values of $a$ and $b$.
(d) Draw a tangent to the graph of $y=\mathrm{f}(x)$ at the point where $x=-2$.

Use it to estimate the gradient of $y=\mathrm{f}(x)$ when $x=-2$.

Answer the whole of this question on a sheet of graph paper.
Use one side for your working and one side for your graphs.

Alaric invests $\$ 100$ at $4 \%$ per year compound interest.
(a) How many dollars will Alaric have after 2 years?
(b) After $x$ years, Alaric will have $y$ dollars.

He knows a formula to calculate $y$.
The formula is $y=100 \times 1.04^{\mathrm{x}}$.

| $x$ (Years) | 0 | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ (Dollars) | 100 | $p$ | 219 | $q$ | 480 |

Use this formula to calculate the values of $p$ and $q$ in the table.
(c) Using a scale of 2 cm to represent 5 years on the $x$-axis and 2 cm to represent $\$ 50$ on the $y$-axis, draw an $x$-axis for $0 \leqslant x \leqslant 40$ and a $y$-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through them.
(d) Use your graph to estimate
(i) how many dollars Alaric will have after 25 years,
(ii) how many years, to the nearest year, it takes for Alaric to have $\$ 200$.
(e) Beatrice invests $\$ 100$ at $7 \%$ per year simple interest.
(i) Show that after 20 years Beatrice has $\$ 240$.
(ii) How many dollars will Beatrice have after 40 years?
(iii) On the same grid, draw a graph to show how the $\$ 100$ which Beatrice invests will increase during the 40 years.
(f) Alaric first has more than Beatrice after $n$ years. Use your graphs to find the value of $n$.

