

# Graphs

## Difficulty: Medium

### Question Paper 4

|            |                   |
|------------|-------------------|
| Level      | IGCSE             |
| Subject    | Maths (0580/0980) |
| Exam Board | CIE               |
| Topic      | Graphs            |
| Paper      | Paper 4           |
| Difficulty | Medium            |
| Booklet    | Question Paper 4  |

**Time allowed:** 86 minutes

**Score:** /75

**Percentage:** /100

#### Grade Boundaries:

##### CIE IGCSE Maths (0580)

| A*   | A   | B   | C   | D   |
|------|-----|-----|-----|-----|
| >83% | 67% | 51% | 41% | 31% |

##### CIE IGCSE Maths (0980)

| 9    | 8   | 7   | 6   | 5   | 4   |
|------|-----|-----|-----|-----|-----|
| >95% | 87% | 80% | 69% | 58% | 46% |

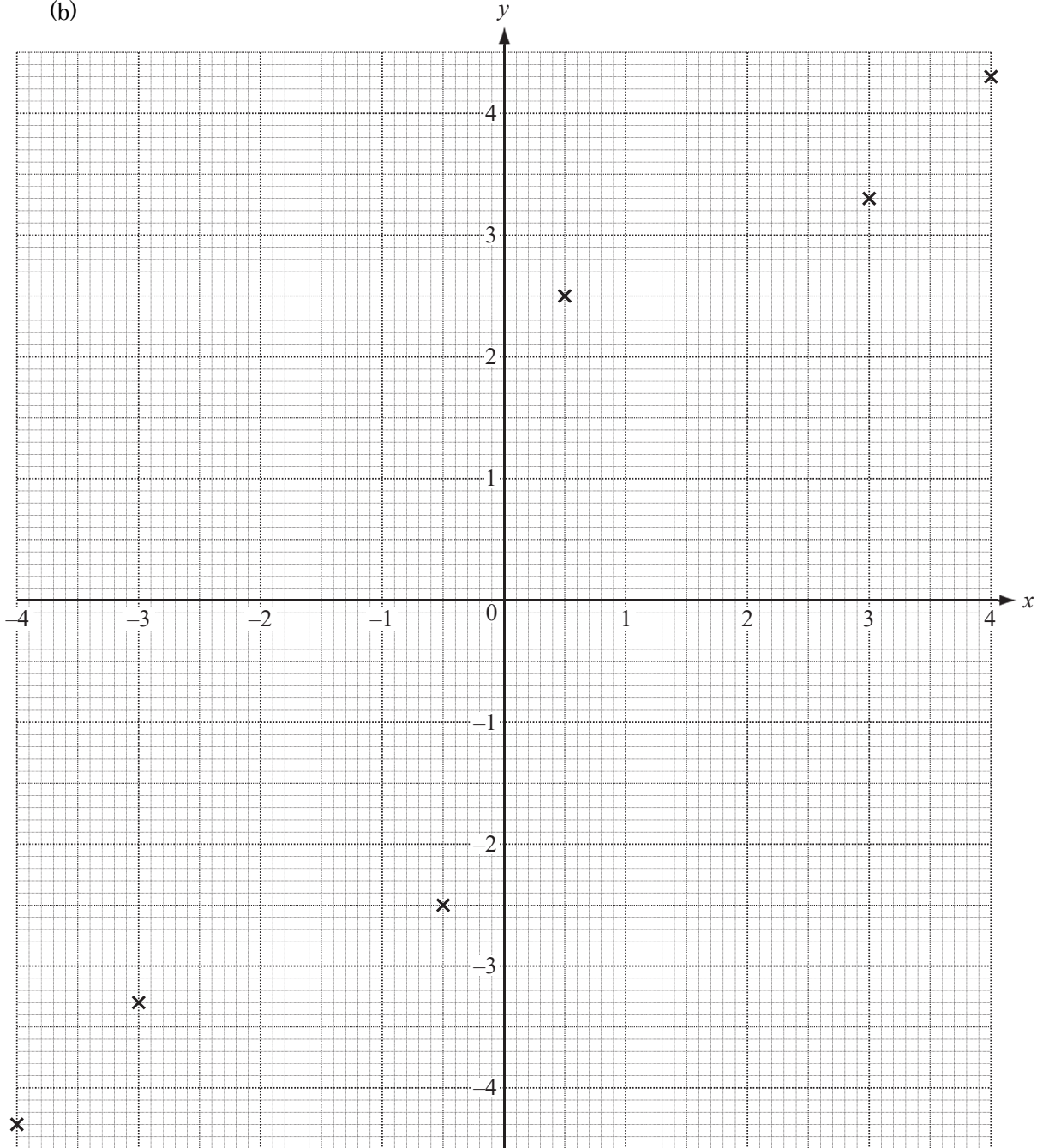
Question 1

(a) Complete the table of values for  $y = x + \frac{1}{x}$ .

|   |      |      |    |    |      |     |   |   |     |     |
|---|------|------|----|----|------|-----|---|---|-----|-----|
| x | -4   | -3   | -2 | -1 | -0.5 | 0.5 | 1 | 2 | 3   | 4   |
| y | -4.3 | -3.3 |    |    | -2.5 | 2.5 |   |   | 3.3 | 4.3 |

[2]

(b)



On the grid, draw the graph of  $y = x + \frac{1}{x}$  for  $-4 \leq x \leq -0.5$  and  $0.5 \leq x \leq 4$ .

Six of the ten points have been plotted for you.

[3]

- (c) There are three integer values of  $k$  for which the equation  $x + \frac{1}{x} = k$  has **no** solutions.  
Write down these three values of  $k$ . [2]

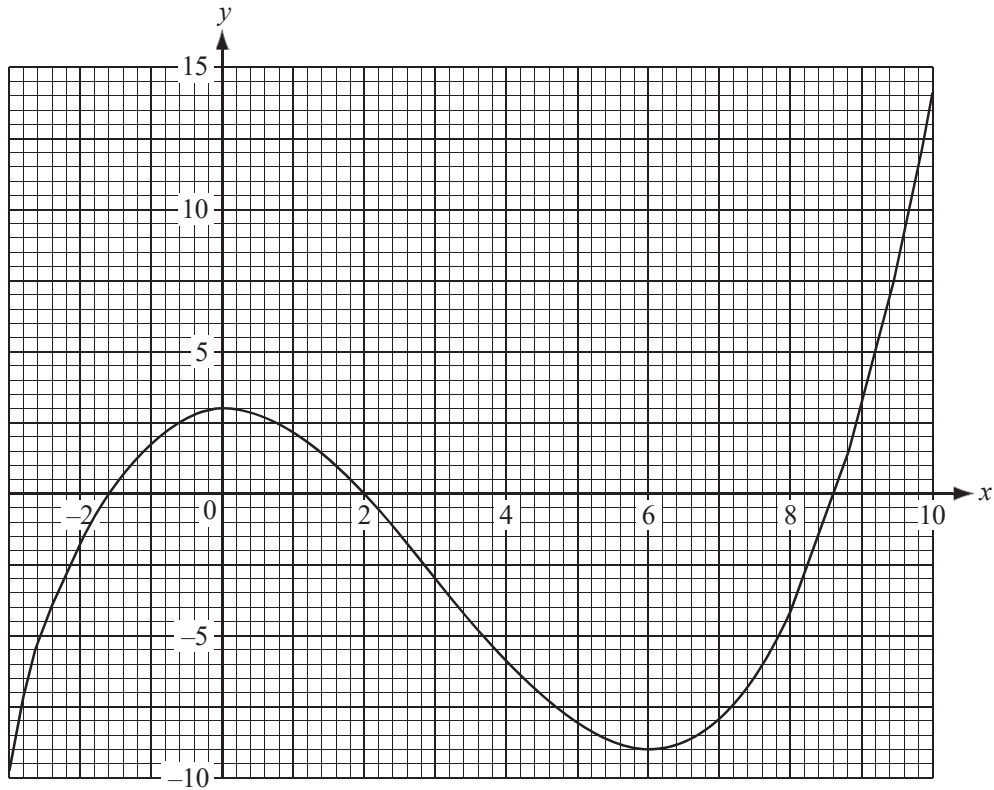
- (d) Write down the ranges of  $x$  for which the gradient of the graph of  $y = x + \frac{1}{x}$  is positive. [2]

- (e) To solve the equation  $x + \frac{1}{x} = 2x + 1$ , a straight line can be drawn on the grid.  
(i) Draw this line on the grid for  $-2.5 \leq x \leq 1.5$ . [2]

- (ii) On the grid, show how you would find the solutions. [1]

- (iii) Show how the equation  $x + \frac{1}{x} = 2x + 1$  can be rearranged into the form  $x^2 + bx + c = 0$   
and find the values of  $b$  and  $c$ . [3]

Question 2



The diagram shows the accurate graph of  $y = f(x)$ .

(a) Use the graph to find

(i)  $f(0)$ , [1]

(ii)  $f(8)$ . [1]

(b) Use the graph to solve

(i)  $f(x) = 0$ , [2]

(ii)  $f(x) = 5$ . [1]

(c)  $k$  is an integer for which the equation  $f(x) = k$  has exactly two solutions.

Use the graph to find the two values of  $k$ . [2]

(d) Write down the range of values of  $x$  for which the graph of  $y = f(x)$  has a negative gradient. [2]

(e) The equation  $f(x) + x - 1 = 0$  can be solved by drawing a line on the grid.

(i) Write down the equation of this line. [1]

(ii) How many solutions are there for  $f(x) + x - 1 = 0$ ? [1]

Answer the whole of this question on a sheet of graph paper.

$$f(x) = 3x - \frac{1}{x^2} + 3, \quad x \neq 0.$$

(a) The table shows some values of  $f(x)$ .

|        |     |      |      |      |    |      |      |      |      |      |     |     |     |     |      |     |
|--------|-----|------|------|------|----|------|------|------|------|------|-----|-----|-----|-----|------|-----|
| $x$    | -3  | -2.5 | -2   | -1.5 | -1 | -0.5 | -0.4 | -0.3 | 0.3  | 0.4  | 0.5 | 1   | 1.5 | 2   | 2.5  | 3   |
| $f(x)$ | $p$ | -4.7 | -3.3 | -1.9 | -1 | -2.5 | -4.5 | -9.0 | -7.2 | -2.1 | 0.5 | $q$ | 7.1 | 8.8 | 10.3 | $r$ |

Find the values of  $p$ ,  $q$  and  $r$ . [3]

(b) Draw axes using a scale of 1 cm to represent 0.5 units for  $-3 \leq x \leq 3$  and 1 cm to represent units for  $-10 \leq y \leq 12$ . [1]

(c) On your grid, draw the graph of  $y = f(x)$  for  $-3 \leq x \leq -0.3$  and  $0.3 \leq x \leq 3$ . [5]

(d) Use your graph to solve the equations

(i)  $3x - \frac{1}{x^2} + 3 = 0$ , [1]

(ii)  $3x - \frac{1}{x^2} + 7 = 0$ . [3]

(e)  $g(x) = 3x + 3$ .

On the same grid, draw the graph of  $y = g(x)$  for  $-3 \leq x \leq 3$ . [2]

(f) (i) Describe briefly what happens to the graphs of  $y = f(x)$  and  $y = g(x)$  for large positive or negative values of  $x$ . [1]

(ii) Estimate the gradient of  $y = f(x)$  when  $x = 100$ . [1]

## Question 4

Answer the whole of this question on a sheet of graph paper.

(a) Find the values of  $k$ ,  $m$  and  $n$  in each of the following equations, where  $a > 0$ .

(i)  $a^0 = k$ , [1]

(ii)  $a^m = \frac{1}{a}$ , [1]

(iii)  $a^n = \sqrt{a^{-3}}$ . [1]

(b) The table shows some values of the function  $f(x) = 2^x$ .

|        |     |     |      |     |      |   |      |   |     |
|--------|-----|-----|------|-----|------|---|------|---|-----|
| $x$    | -2  | -1  | -0.5 | 0   | 0.5  | 1 | 1.5  | 2 | 3   |
| $f(x)$ | $r$ | 0.5 | 0.71 | $s$ | 1.41 | 2 | 2.83 | 4 | $t$ |

(i) Write down the values of  $r$ ,  $s$  and  $t$ . [3]

(ii) Using a scale of 2 cm to represent 1 unit on each axis, draw an  $x$ -axis from -2 to 3 and a  $y$ -axis from 0 to 10. [1]

(iii) On your grid, draw the graph of  $y = f(x)$  for  $-2 \leq x \leq 3$ . [4]

(c) The function  $g$  is given by  $g(x) = 6 - 2x$ .

(i) On the same grid as **part (b)**, draw the graph of  $y = g(x)$  for  $-2 \leq x \leq 3$ . [2]

(ii) Use your graphs to solve the equation  $2 = 6 - 2x$ . [1]

(iii) Write down the value of  $x$  for which  $2 < 6 - 2x$  for  $x \in \{\text{positive integers}\}$ . [1]

## Question 5

Answer the whole of this question on a sheet of graph paper.

The table gives values of  $f(x) = 2^x$ , for  $-2 \leq x \leq 4$ .

|        |     |     |     |   |   |     |    |
|--------|-----|-----|-----|---|---|-----|----|
| $x$    | -2  | -1  | 0   | 1 | 2 | 3   | 4  |
| $f(x)$ | $p$ | 0.5 | $q$ | 2 | 4 | $r$ | 16 |

(a) Find the values of  $p$ ,  $q$  and  $r$ . [3]

(b) Using a scale of 2 cm to 1 unit on the  $x$ -axis and 1 cm to 1 unit on the  $y$ -axis, draw the graph of  $y = f(x)$  for  $-2 \leq x \leq 4$ . [5]

(c) Use your graph to solve the equation  $2^x = 7$ . [1]

(d) What value does  $f(x)$  approach as  $x$  decreases? [1]

(e) By drawing a tangent, estimate the gradient of the graph of  $y = f(x)$  when  $x = 1.5$ . [3]

(f) On the same grid draw the graph of  $y = 2x + 1$  for  $0 \leq x \leq 4$ . [2]

(g) Use your graph to find the non-integer solution of  $2^x = 2x + 1$ . [2]