## Graphs <br> Difficulty: Medium

## Question Paper 3

| Level | IGCSE |
| :--- | :--- |
| Subject | Maths (0580/0980) |
| Exam Board | CIE |
| Topic | Graphs |
| Paper | Paper 4 |
| Difficulty | Medium |
| Booklet | Question Paper 3 |


| Time allowed: | $\mathbf{1 2 2}$ minutes |
| :--- | :--- |
| Score: | $/ 106$ |
| Percentage: | $/ 100$ |

## Grade Boundaries:

CIE IGCSE Maths (0580)

| A* | A | B | C | D |
| :--- | :--- | :--- | :--- | :--- |
| $>83 \%$ | $67 \%$ | $51 \%$ | $41 \%$ | $31 \%$ |

CIE IGCSE Maths (0980)

| 9 | 8 | 7 | 6 | 5 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $>95 \%$ | $87 \%$ | $80 \%$ | $69 \%$ | $58 \%$ | $46 \%$ |

The table shows some values for the equation $y=x^{3}-2 x$ for $-2 \leqslant x \leqslant 2$.

| $x$ | -2 | -1.5 | -1 | -0.6 | -0.3 | 0 | 0.3 | 0.6 | 1 | 1.5 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -4 | -0.38 |  |  | 0.57 |  | -0.57 |  |  | 0.38 | 4 |

(a) Complete the table of values.
(b) On the grid below, draw the graph of $y=x^{3}-2 x$ for $-2 \leqslant x \leqslant 2$.

The first two points have been plotted for you.

(c) (i) On the grid, draw the line $y=0.8$ for $-2 \leqslant x \leqslant 2$.
(ii) Use your graph to solve the equation $x^{3}-2 x=0.8$.
(d) By drawing a suitable tangent, work out an estimate for the gradient of the graph of $y=x^{3}-2 x$ where $x=-1.5$.

You must show your working.
(a) Complete the table for the function $\mathrm{f}(x)=\frac{\mathrm{x}^{3}}{2}-3 \mathrm{x}-1$.

| $x$ | -3 | -2 | -1.5 | -1 | 0 | 1 | 1.5 | 2 | 3 | 3.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | -5.5 |  | 1.8 | 1.5 |  | -3.5 | -3.8 | -3 |  | 9.9 |

(b) On the grid draw the graph of $y=\mathrm{f}(x)$ for $-3 \leq x \leq 3.5$

(c) Use your graph to
(i) solve $\mathrm{f}(x)=0.5$,
(ii) find the inequalities for $k$, so that $\mathrm{f}(x)=k$ has only 1 answer.
(d) (i) On the same grid, draw the graph of $y=3 x-2$ for $-1 \leq x \leq 3.5$
(ii) The equation $\frac{\mathrm{x}^{3}}{2}-3 \mathrm{x}-1=3 x-2$ can be written in the form $x^{3}+a x+b=0$. Find the values of $a$ and $b$.
(iii) Use your graph to find the positive answers to $\frac{x^{3}}{2}-3 x-1=3 x-2$ for $-3 \leq x \leq 3.5$.

The diagram shows the accurate graph of $y=\mathrm{f}(x)$ where $\mathrm{f}(x)=\frac{1}{\mathrm{x}}+x^{2}$ for $0<x \leqslant 3$.

(a) Complete the table for $\mathrm{f}(x)=\frac{1}{\mathrm{x}}+$. $^{2}$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.3 | -0.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ |  | 3.5 | 0 | -1.8 |  |  |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x<0$.
(c) By drawing a tangent, work out an estimate of the gradient of the graph where $x=2$.
(d) Write down the inequality satisfied by $k$ when $\mathrm{f}(x)=k$ has three answers.
(e) (i) Draw the line $y=1-x$ on the grid for $-3 \leqslant x \leqslant 3$.
(ii) Use your graphs to solve the equation $1-x=\frac{1}{\mathrm{x}}+x^{2}$.
(f) (i) Rearrange $x^{3}-x-2 x+1=0$ into the form $\frac{1}{\mathrm{x}}+x^{2}=a x+b$, where $a$ and $b$ are integers.
(ii) Write down the equation of the line that could be drawn on the graph
to solve $x^{3}-x^{2}-2 x+1=0$.
(a) Complete the table of values for $y=2^{x}$.
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| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0.25 |  | 1 | 2 |  | 8 |

(b) On the grid, draw the graph of $y=2^{x}$ for $-2 \leqslant x \leqslant 3$.

(c) (i) On the grid, draw the straight line which passes through the points $(0,2)$ and $(3,8)$.
(ii) The equation of this line is $y=m x+2$.

Show that the value of $m$ is 2 .
(iii) One answer to the equation $2^{x}=2 x+2$ is $x=3$.

Use your graph to find the other answer.
(d) Draw the tangent to the curve at the point where $x=1$.

Use this tangent to calculate an estimate of the gradient of $y=2^{x}$ when $x=1$.


A farmer makes a rectangular enclosure for his animals.
He uses a wall for one side and a total of 72 metres of fencing for the other three sides.
The enclosure has width $x$ metres and area $A$ square metres.
(a) Show that $A=72 x-2 x .^{2}$
(b) Factorise completely $72 x-2 x^{2}$.
(c) Complete the table for $A=72 x-2 x^{2}$.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | 0 | 310 | 520 |  |  | 550 | 360 |  |

(d) Draw the graph of $A=72 x-2 x^{2}$ for $0 \leqslant x \leqslant 35$ on the grid opposite.

(e) Use your graph to find
(i) the values of $x$ when $A=450$,
(ii) the maximum area of the enclosure.
[1]
(f) Each animal must have at least $12 \mathrm{~m}^{2}$ for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of $500 \mathrm{~m}^{2}$.
(a) Complete the table for the function $\mathrm{f}(x)=\frac{x^{3}}{10}+1$

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ |  | -1.7 | 0.2 | 0.9 | 1 | 1.1 | 1.8 |  |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-4 \leqslant x \leqslant 3$.

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(c) Complete the table for the function $\mathrm{g}(x)=\frac{4}{x}, x \neq 0$.

| $x$ | -4 | -3 | -2 | -1 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}(x)$ | -1 | -1.3 |  |  |  | 2 | 1.3 |

(d) On the grid, draw the graph of $y=\mathrm{g}(x)$ for $-4 \leqslant x \leqslant-1$ and $1 \leqslant x \leqslant 3$.
(e) (i) Use your graphs to solve the equation $\frac{\mathrm{x}^{3}}{10}+1=\frac{4}{\mathrm{x}}$.
(ii) The equation $\frac{\mathrm{x}^{3}}{10}+\mathrm{I}=\frac{4}{\mathrm{x}} \quad$ can be written as $x^{4}+a x+b=0$.

Find the values of $a$ and $b$.
(a) $\mathrm{f}(x)=2$

Complete the table.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=\mathrm{f}(x)$ |  | 0.5 | 1 | 2 | 4 |  |  |

(b) $\mathrm{g}(x)=x(4-x)$

Complete the table.

| $x$ | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=\mathrm{g}(x)$ |  | 0 | 3 |  | 3 | 0 |

[2]
(c) On the grid, draw the graphs of
(i) $y=\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 4$,
(ii) $y=\mathrm{g}(x)$ for $-1 \leqslant x \leqslant 4$.

(d) Use your graphs to solve the following equations.
(i) $\mathrm{f}(x)=10$
(ii) $\mathrm{f}(x)=\mathrm{g}(x)$
(iii) $\mathrm{f}^{-1}(x)=1.7$

