Loci
Difficulty: Medium

Question Paper 2

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Complex Numbers
Sub-Topic	Loci
Difficulty	Medium
Booklet	Question Paper 2

Time allowed: **57 minutes**

Score: **/41**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Question 1

The complex number $\frac{2}{-1+i}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z - u^2| < |z - u|$. [4]

Question 2

(a) The complex numbers u and w satisfy the equations

$$u - w = 4\mathrm{i} \quad \text{and} \quad uw = 5.$$

Solve the equations for u and w, giving all answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 + 2\mathrm{i}| \leq 2$, $\arg z \leq -\frac{1}{4}\pi$ and $\mathrm{Re}\, z \geq 1$, where $\mathrm{Re}\, z$ denotes the real part of z. [5]

(ii) Calculate the greatest possible value of $\mathrm{Re}\, z$ for points lying in the shaded region. [1]

(a) The complex number w is such that $\text{Re}\, w > 0$ and $w + 3w^* = \text{i}\, w^2$, where w^* denotes the complex conjugate of w. Find w, giving your answer in the form $x + \text{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2\text{i}| \leq 2$ and $0 \leq \arg(z + 2) \leq \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

Question 4

The complex number $1 - i$ is denoted by u.

(i) Showing your working and without using a calculator, express

$$\frac{i}{u}$$

in the form $x + iy$, where x and y are real. [2]

(ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - i| = 2$. [4]

(iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part **(ii)**. [3]