Loci
Difficulty: Easy

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Complex Numbers
Sub-Topic	Loci
Difficulty	Easy
Booklet	Question Paper 1

Time allowed: **48 minutes**

Score: **/34**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Throughout this question the use of a calculator is not permitted.

The complex number $2 - i$ is denoted by u.

(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

(ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + i|$. [4]

The variable complex number z is given by

$$z = 2\cos\theta + i(1 - 2\sin\theta),$$

where θ takes all values in the interval $-\pi < \theta \leq \pi$.

(i) Show that $|z - i| = 2$, for all values of θ. Hence sketch, in an Argand diagram, the locus of the point representing z. [3]

(ii) Prove that the real part of $\dfrac{1}{z + 2 - i}$ is constant for $-\pi < \theta < \pi$. [4]

The complex number 2 + 2i is denoted by u.

(i) Find the modulus and argument of u. [2]

(ii) Sketch an Argand diagram showing the points representing the complex numbers 1, i and u. Shade the region whose points represent the complex numbers z which satisfy both the inequalities

$$|z - 1| \leqslant |z - \mathrm{i}| \text{ and } |z - u| \leqslant 1.$$

[4]

(iii) Using your diagram, calculate the value of |z| for the point in this region for which arg z is least.

[3]

Question 4

The complex number u is defined by $u = \frac{6 - 3i}{1 + 2i}$

(i) Showing all your working, find the modulus of u and show that the argument of u is $-\frac{1}{2}\pi$. [4]

(ii) For complex numbers z satisfying $\arg(z - u) = \frac{1}{4}\pi$, find the least possible value of $|z|$. [3]

(iii) For complex numbers z satisfying $|z - (1 + i)u| = 1$, find the greatest possible value of $|z|$. [3]