Polars
Difficulty: Easy

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Complex Numbers
Sub-Topic	Polars
Difficulty	Easy
Booklet	Question Paper 1

Time allowed: **36 minutes**

Score: **/26**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Throughout this question the use of a calculator is not permitted.

The complex numbers $-1 + 3i$ and $2 - i$ are denoted by u and v respectively. In an Argand diagram with origin O, the points A, B and C represent the numbers u, v and $u + v$ respectively.

(i) Sketch this diagram and state fully the geometrical relationship between OB and AC. [4]

(ii) Find, in the form $x + iy$, where x and y are real, the complex number $\frac{u}{v}$. [3]

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

The complex number $2 + \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) Show, on a sketch of an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u + u^*$ respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

(ii) Express $\frac{u}{u^*}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u}{u^*}$, or otherwise, prove that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right).$$ [2]

Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{1+2i}{1-3i}.$$

(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Show on a sketch of an Argand diagram the points A, B and C representing the complex numbers u, $1 + 2i$ and $1 - 3i$ respectively. [2]

(iii) By considering the arguments of $1 + 2i$ and $1 - 3i$, show that

$$\tan^{-1} 2 + \tan^{-1} 3 = \tfrac{3}{4}\pi.$$ [3]