

Rates of Change

Difficulty: Medium

Question Paper 1

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Differentiation
Sub-Topic	Rates of Change
Difficulty	Medium
Booklet	Question Paper 1

Time allowed: 48 minutes

Score: /40

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

Question 1

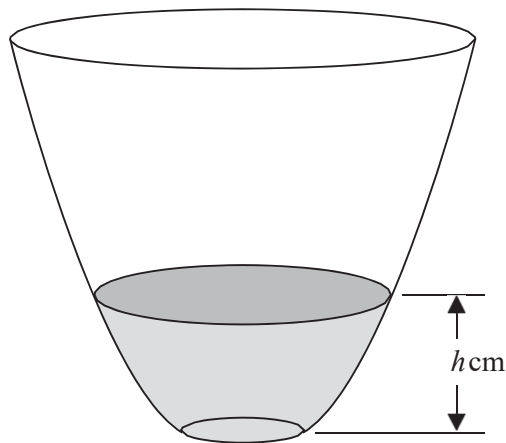


Figure 2

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm³ is given by

$$V = 4\pi h(h + 4), \quad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π cm³s⁻¹

Find the rate of change of the depth of the water, in cms⁻¹, when $h = 6$ **(5)**

(Total 5 marks)

Question 2

The volume of a sphere V cm³ is related to its radius r cm by the formula $V = \frac{4}{3}\pi r^3$. The surface area of the sphere is also related to the radius by the formula $S = 4\pi r^2$. Given that the rate of decrease in

surface area, in cm²s⁻¹, is $\frac{dS}{dt} = -12$,

find the rate of decrease of volume $\frac{dV}{dt}$ (4)

(Total 4 marks)

Question 3

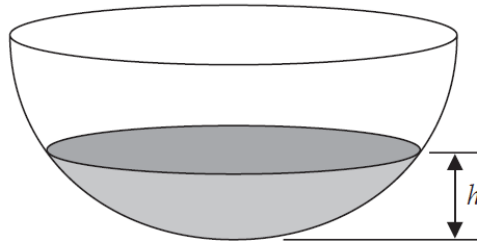


Figure 3

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is h cm, the volume of water, V cm³, according to the model is given by

$$V = \frac{1}{3} \pi h^2 (75 - h), \quad 0 \leq h \leq 24$$

The flow of water into the bowl is at a constant rate of 160π cm³ s⁻¹ for $0 \leq h \leq 12$

(a) Find the rate of change of the depth of the water, in cm s⁻¹, when $h = 10$

(5)

Given that the flow of water into the bowl is increased to a constant rate of 300π cm³ s⁻¹ for $12 < h \leq 24$

(b) find the rate of change of the depth of the water, in cm s⁻¹, when $h = 20$

(2)

(Total 7 marks)

Question 4

At time t seconds the radius of a sphere is r cm, its volume is V cm³ and its surface area is S cm².

$$\left[\text{You are given that } V = \frac{4}{3}\pi r^3 \text{ and that } S = 4\pi r^2 \right]$$

The volume of the sphere is increasing uniformly at a constant rate of $3 \text{ cm}^3 \text{ s}^{-1}$.

- (a) Find $\frac{dr}{dt}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures. (4)

- (b) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm. (2)

(Total 6 marks)

Question 5

A circular stain grows in such a way that the rate of increase of its radius is inversely proportional to the square of the radius. Given that the area of the stain at time t seconds is $A \text{ cm}^2$,

(a) show that $\frac{dA}{dt} \propto \frac{1}{\sqrt{A}}$. (6)

(Total 6 marks)

Question 6

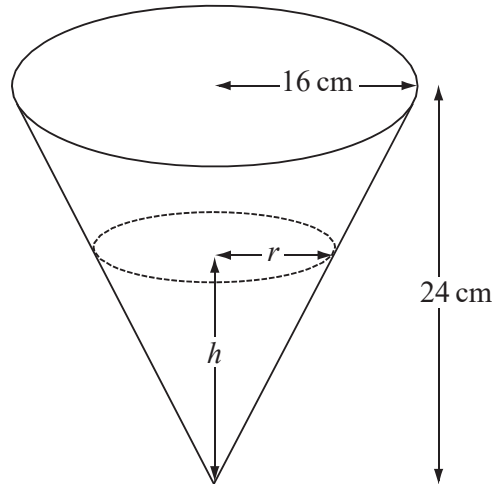


Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm³.

(a) Show that $V = \frac{4\pi h^3}{27}$ (2)

[The volume V of a right circular cone with vertical height h and base radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.]

Water flows into the container at a rate of $8 \text{ cm}^3 \text{ s}^{-1}$.

(b) Find, in terms of π , the rate of change of h when $h = 12$. (5)

(Total 7 marks)

Question 7

Oil is leaking from a storage container onto a flat section of concrete at a rate of $0.48 \text{ cm}^3 \text{ s}^{-1}$. The leaking oil spreads to form a pool with an increasing circular cross-section. The pool has a constant uniform thickness of 3 mm.

Find the rate at which the radius r of the pool of oil is increasing at the instant when $r = 5 \text{ cm}$. Give your answer, in cm s^{-1} , to 3 significant figures.

(5)

(Total 5 marks)