

Separation of Variables

Difficulty: Medium

Question Paper 5

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Differential Equations
Sub-Topic	Separation of Variables
Difficulty	Medium
Booklet	Question Paper 5

Time allowed: 57 minutes

Score: /41

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Question 1

The variables x and y satisfy the differential equation

$$(x + 1) \frac{dy}{dx} = y(x + 2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x . [7]

Question 2

The variables x and θ satisfy the differential equation

$$x \cos^2 \theta \frac{dx}{d\theta} = 2 \tan \theta + 1,$$

for $0 \leq \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 1$ when $\theta = \frac{1}{4}\pi$.

(i) Show that $\frac{d}{d\theta}(\tan^2 \theta) = \frac{2 \tan \theta}{\cos^2 \theta}$. [1]

(ii) Solve the differential equation and calculate the value of x when $\theta = \frac{1}{3}\pi$ giving your answer correct to 3 significant figures. [7]

Question 3

(i) Express $\frac{1}{4-y^2}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x \frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x . [6]

Question 4

A certain curve is such that its gradient at a general point with coordinates (x, y) is proportional to $\frac{y^2}{x}$. The curve passes through the points with coordinates $(1, 1)$ and $(e, 2)$. By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x . [8]

Question 5

The number of insects in a population t days after the start of observations is denoted by N . The variation in the number of insects is modelled by a differential equation of the form

$$\frac{dN}{dt} = kN \cos(0.02t),$$

where k is a constant and N is taken to be a continuous variable. It is given that $N = 125$ when $t = 0$.

(i) Solve the differential equation, obtaining a relation between N , k and t . [5]

(ii) Given also that $N = 166$ when $t = 30$, find the value of k . [2]

(iii) Obtain an expression for N in terms of t , and find the least value of N predicted by this model. [3]