Solving Differential Equations
Difficulty: Easy

Question Paper 2

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Differential Equations
Sub-Topic	Solving Differential Equations
Difficulty	Easy
Booklet	Question Paper 2

Time allowed: **45 minutes**

Score: **/32**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Question 1

Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{x} - \frac{x}{4},$$

obtaining an expression for x^2 in terms of t. [7]

Question 2

Given that $y = 2$ when $x = 0$, solve the differential equation

$$y\frac{dy}{dx} = 1 + y^2,$$

obtaining an expression for y^2 in terms of x. [6]

The temperature of a quantity of liquid at time t is θ. The liquid is cooling in an atmosphere whose temperature is constant and equal to A. The rate of decrease of θ is proportional to the temperature difference $(\theta - A)$. Thus θ and t satisfy the differential equation

$$\frac{d\theta}{dt} = -k(\theta - A),$$

where k is a positive constant.

(i) Find, in any form, the solution of this differential equation, given that $\theta = 4A$ when $t = 0$. [5]

(ii) Given also that $\theta = 3A$ when $t = 1$, show that $k = \ln \frac{3}{2}$. [1]

(iii) Find θ in terms of A when $t = 2$, expressing your answer in its simplest form. [3]

A biologist is investigating the spread of a weed in a particular region. At time t weeks after the start of the investigation, the area covered by the weed is A m^2. The biologist claims that the rate of increase of A is proportional to $\sqrt{(2A - 5)}$.

(i) Write down a differential equation representing the biologist's claim. [1]

(ii) At the start of the investigation, the area covered by the weed was 7 m^2 and, 10 weeks later, the area covered was 27 m^2. Assuming that the biologist's claim is correct, find the area covered 20 weeks after the start of the investigation. [9]