Solving Differential Equations
Difficulty: Easy

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Differential Equations
Sub-Topic	Solving Differential Equations
Difficulty	Easy
Booklet	Question Paper 1

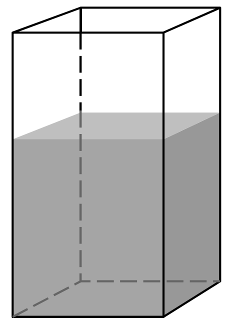
Time allowed: **49 minutes**

Score: **/35**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%



A water tank has vertical sides and a horizontal rectangular base, as shown in the diagram. The area of the base is 2 m^2. At time $t = 0$ the tank is empty and water begins to flow into it at a rate of 1 m^3 per hour. At the same time water begins to flow out from the base at a rate of $0.2\sqrt{h}$ m^3 per hour, where h m is the depth of water in the tank at time t hours.

(i) Form a differential equation satisfied by h and t, and show that the time T hours taken for the depth of water to reach 4 m is given by

[3]

$$T = \int_0^4 \frac{10}{5 - \sqrt{h}}\,dh.$$

(ii) Using the substitution $u = 5 - \sqrt{h}$, find the value of T. [6]

In a certain chemical reaction, a compound A is formed from a compound B. The masses of A and B at time t after the start of the reaction are x and y respectively and the sum of the masses is equal to 50 throughout the reaction. At any time the rate of increase of the mass of A is proportional to the mass of B at that time.

(i) Explain why $\frac{dx}{dt} = k(50 - x)$, where k is a constant. [1]

It is given that $x = 0$ when $t = 0$, and $x = 25$ when $t = 10$.

(ii) Solve the differential equation in part (i) and express x in terms of t. [8]

(i) Using partial fractions, find

$$\int \frac{1}{y(4-y)}\,\mathrm{d}y.$$ [4]

(ii) Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y(4-y),$$

obtaining an expression for y in terms of x. [4]

(iii) State what happens to the value of y if x becomes very large and positive. [1]

In a certain industrial process, a substance is being produced in a container. The mass of the substance in the container t minutes after the start of the process is x grams. At any time, the rate of formation of the substance is proportional to its mass. Also, throughout the process, the substance is removed from the container at a constant rate of 25 grams per minute. When $t = 0$, $x = 1000$ and $\frac{dx}{dt} = 75$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.1(x - 250).$$ [2]

(ii) Solve this differential equation, obtaining an expression for x in terms of t. [6]