Scalar Product
Difficulty: Medium

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Vectors
Sub-Topic	Scalar Product
Difficulty	Medium
Booklet	Question Paper 1

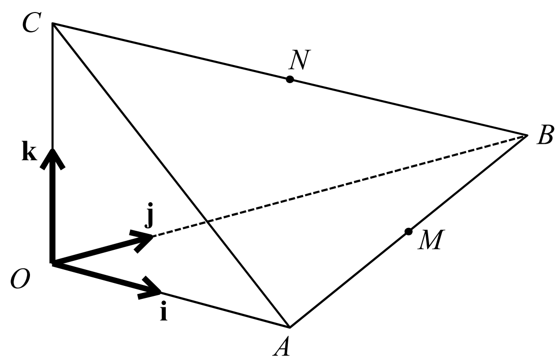
Time allowed: **38 minutes**

Score: **/27**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%



In the diagram, $OABC$ is a pyramid in which $OA = 2$ units, $OB = 4$ units and $OC = 2$ units. The edge OC is vertical, the base OAB is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC respectively. The midpoints of AB and BC are M and N respectively.

(a) Express the vectors $\overrightarrow{ON}$ and $\overrightarrow{CM}$ in terms of **i**, **j** and **k**. [3]

(b) Calculate the angle between the directions of $\overrightarrow{ON}$ and $\overrightarrow{CM}$. [3]

(c) Show that the length of the perpendicular from M to ON is $\frac{3}{5}\sqrt{5}$. [4]

The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

(i) Show that l does not intersect the line passing through A and B. [4]

(ii) The point P lies on l and is such that angle PAB is equal to 60°. Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P. [6]

Referred to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

(i) Find the exact value of the cosine of angle BAC. [4]

(ii) Hence find the exact value of the area of triangle ABC. [3]