Addition & Double Angle formulae Difficulty: Medium

Question Paper 1

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Trigonometry & Modelling
Sub-Topic	Addition & Double Angle Formulae
Difficulty	Medium
Booklet	Question Paper 1

Time allowed: 67 minutes

Score: /56

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

(a) Given that $\cos A = \frac{3}{4}$, where $270° < A < 360°$, find the exact value of $\sin 2A$.

(5)

(b) (i) Show that $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right) \equiv \cos 2x$.

Given that

$$y = 3\sin^2 x + \cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right),$$

(3)

(Total 8 marks)

(a) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2\operatorname{cosec} 2\theta, \qquad \theta \neq 90n°.$$ **(4)**

(b) Sketch the graph of $y = 2\operatorname{cosec} 2\theta$ for $0° < \theta < 360°$. **(2)**

(c) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3,$$

giving your answers to 1 decimal place. **(6)**

(Total 12 marks)

(a) (i)   By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \dfrac{\pi}{3}$, solve

$$8 \sin^3\theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)

(Total 13 marks)

(a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan \theta, \quad \theta \neq 90n°, \ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

(5)

(Total 12 marks)

(i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2$$

You must show each stage of your working. **(5)**

(ii) (a) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k. \qquad \textbf{(2)}$$

(b) Hence solve, for $0 \leqslant \theta < 360°$, the equation

$$\cos 2\theta + \sin \theta = 1 \qquad \textbf{(4)}$$

(Total 11 marks)