Iterative Methods
Difficulty: Easy
Question Paper 3

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Numerical Solutions
Sub-Topic	Iterative Methods
Difficulty	Easy
Booklet	Question Paper 3

Time allowed: **52 minutes**

Score: **/37**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

(i) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

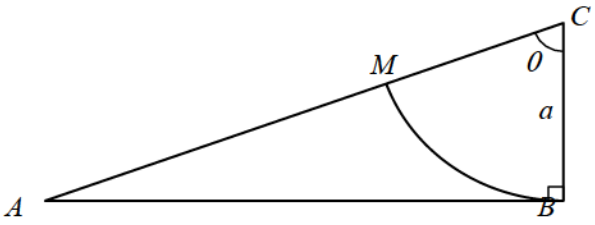
where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 0.8. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{1 + x_n^2}\right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]



In the diagram, ABC is a triangle in which angle ABC is a right angle and $BC = a$. A circular arc, with centre C and radius a, joins B and the point M on AC. The angle ACB is θ radians. The area of the sector CMB is equal to one third of the area of the triangle ABC.

(i) Show that θ satisfies the equation

$$\tan\theta = 3\theta.$$ [2]

(ii) This equation has one root in the interval $0 < \theta < \frac{1}{2}\pi$. Use the iterative formula

$$\theta_{n+1} = \tan^{-1}(3\theta_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

The sequence of values given by the iterative formula

$$x_{n+1} = \frac{x_n(x_n^3 + 100)}{2(x_n^3 + 25)},$$

with initial value $x_1 = 3.5$, converges to α .

(i) Use this formula to calculate α correct to 4 decimal places, showing the result of each iteration to 6 decimal places. [3]

.

(ii) State an equation satisfied by α and hence find the exact value of α [2]

The equation $x = \dfrac{10}{e^{2x} - 1}$ has one positive real root, denoted by α.

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2} \ln\left(1 + \frac{10}{x_n}\right)$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α.

(i) Find by calculation the pair of consecutive integers between which α lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

The equation $x^3 = 3x + 7$ has one real root, denoted by α.

(i) Show by calculation that α lies between 2 and 3. [2]

Two iterative formulae, A and B, derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}} \qquad (A)$$

$$x_{n+1} = \frac{x_n^3 - 7}{3}. \qquad (B)$$

Each formula is used with initial value $x_1 = 2.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate **a** correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [4]