Iterative Methods
Difficulty: Medium

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Numerical Solutions
Sub-Topic	Iterative Methods
Difficulty	Medium
Booklet	Question Paper 1

Time allowed: **83 minutes**

Score: **/59**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

The parametric equations of a curve are

$$x = e^{2t-3}, \quad y = 4\ln t,$$

where $t > 0$. When $t = a$ the gradient of the curve is 2.

(a) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [4]

(b) Verify by calculation that this equation has a root between 1 and 2. [2]

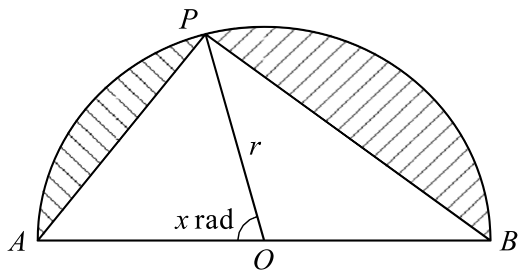
(c) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

The curve with equation $y = x^2 \cos \frac{1}{2}x$ has a stationary point at $x = p$ in the interval $0 < x < \pi$.

(i) Show that p satisfies the equation $\tan \frac{1}{2} p = \frac{4}{p}$ [3]

(ii) Verify by calculation that p lies between 2 and 2.5. [2]

(iii) Use the iterative formula $p_{n+1} = 2 \tan^{-1}\left(\frac{4}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]



The diagram shows a semicircle with centre O, radius r and diameter AB. The point P on its circumference is such that the area of the minor segment on AP is equal to half the area of the minor segment on BP. The angle AOP is x radians.

(i) Show that x satisfies the equation $x = \frac{1}{3}(\pi + \sin x)$. [3]

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use an iterative formula based on the equation in part (i) to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

Question 4

In a chemical reaction a compound X is formed from a compound Y. The masses in grams of X and Y present at time t seconds after the start of the reaction are x and y respectively. The sum of the two masses is equal to 100 grams throughout the reaction. At any time, the rate of formation of X is proportional to the mass of Y at that time. When $t = 0$, $x = 5$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 1.9$.

(i) Show that x satisfies the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.02(100 - x).$$ [2]

(ii) Solve this differential equation, obtaining an expression for x in terms of t. [6]

(iii) State what happens to the value of x as t becomes very large. [1]

(i) By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec} x = \tfrac{1}{2}x + 1,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify, by calculation, that this root lies between 0.5 and 1. [2]

(iii) Show that this root also satisfies the equation

$$x = \sin^{-1}\left(\frac{2}{x+2}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2}{x_n + 2}\right),$$

with initial value $x_1 = 0.75$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(i) By sketching a suitable pair of graphs, show that the equation

$$2\cot x = 1 + e^x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1.0. [2]

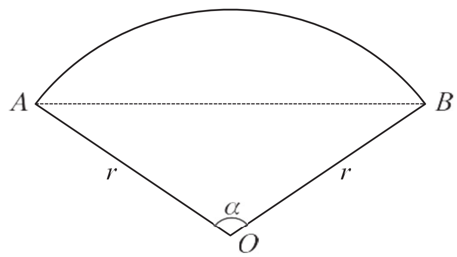
(iii) Show that this root also satisfies the equation

$$x = \tan^{-1}\left(\frac{2}{1 + e^x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{2}{1 + e^{x_n}}\right),$$

with initial value $x_1 = 0.7$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]



The diagram shows a sector AOB of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of triangle AOB is half the area of the sector.

(i) Show that α satisfies the equation

$$x = 2\sin x.$$ [2]

(ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{3}(x_n + 4\sin x_n)$$

converges, then it converges to a root of the equation in part **(i)**. [2]

(iv) Use this iterative formula, with initial value $x_1 = 1.8$, to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]