Iterative Methods
Difficulty: Easy
Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Numerical Solutions
Sub-Topic	Iterative Methods
Difficulty	Easy
Booklet	Question Paper 1

Time allowed: **49 minutes**

Score: **/35**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α .

(i) Find by calculation the pair of consecutive integers between which a lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α . [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

The equation $x^5 - 3x^3 + x^2 - 4 = 0$ has one positive root.

(i) Verify by calculation that this root lies between 1 and 2. [2]

(ii) Show that the equation can be rearranged in the form

$$x = \sqrt[3]{\left(3x + \frac{4}{x^2} - 1\right)}.$$ [1]

(iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(i) By sketching suitable graphs, show that the equation $e^{-\frac{1}{2}x} = 4 - x^2$ has one positive root and one negative root. [2]

(ii) Verify by calculation that the negative root lies between -1 and -1.5. [2]

(iii) Use the iterative formula $x_{n+1} = -\sqrt{(4 - e^{-\frac{1}{2}x_n})}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(i) By sketching a suitable pair of graphs, show that the equation

$$5e^{-x} = \sqrt{x}$$

has one root. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{25}{x_n}\right)$$

converges, then it converges to the root of the equation in part (i). [2]

(iii) Use this iterative formula, with initial value $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Question 5

The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{dx}{d\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

(i) State the least value taken by x. [1]