

# Integration By Substitution

## Difficulty: Medium

### Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Integration
Sub-Topic	Integration by Substitution
Difficulty	Medium
Booklet	Question Paper 1

**Time allowed:** 57 minutes

**Score:** /41

**Percentage:** /100

#### Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

## Question 1

It is given that  $x = \ln(1 - y) - \ln y$ , where  $0 < y < 1$ .  $e^{-x}$

(i) Show that  $y = \frac{1}{1 + e^{-x}}$ . [2]

(ii) Hence show that  $\int_0^1 y \, dx = \ln\left(\frac{2e}{e+1}\right)$ . [4]

## Question 2

$$\text{Let } I = \int_1^4 \frac{1}{x(4 - \sqrt{x})} dx.$$

(i) Use the substitution  $u = \sqrt{x}$  to show that  $I = \int_1^2 \frac{2}{u(4 - u)} du$ . [3]

(ii) Hence show that  $I = \frac{1}{2} \ln 3$ . [6]

### Question 3

The integral  $I$  is defined by  $I = \int_0^2 4t^3 \ln(t^2 + 1) dt$ .

(i) Use the substitution  $x = t^2 + 1$  to show that  $I = \int_1^5 (2x - 2) \ln x dx$ . [3]

(ii) Hence find the exact value of  $I$ . [5]

## Question 4

$$\text{Let } I = \int_2^5 \frac{5}{x + \sqrt{6-x}} dx.$$

(i) Using the substitution  $u = \sqrt{6-x}$ , show that

$$I = \int_1^2 \frac{10u}{(3-u)(2+u)} du. \quad [4]$$

(ii) Hence show that  $I = 2 \ln \left(\frac{9}{2}\right)$ . [6]

## Question 5

$$\text{Let } I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}} dx.$$

(i) Using the substitution  $u = 2 - \sqrt{x}$ , show that  $I = \int_1^2 \frac{2(2-u)^2}{u} du$ . [4]

(ii) Hence show that  $I = 8 \ln 2 - 5$ . [4]