

Integration using Partial Fractions

Difficulty: Medium

Question Paper 3

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Integration
Sub-Topic	Integration using Partial Fractions
Difficulty	Medium
Booklet	Question Paper 3

Time allowed: 50 minutes

Score: /36

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Question 1

$$\text{Let } f(x) = \frac{5x^2 + x + 27}{(2x + 1)(x^2 + 9)}.$$

(i) Express $f(x)$ in partial fractions.

[5]

(ii) Hence find $\int_0^4 f(x) \, dx$, giving your answer in the form $\ln c$, where c is an integer.

[5]

Question 2

$$\text{Let } f(x) = \frac{6x^2 + 8x + 9}{(2-x)(3+2x)^2}.$$

(i) Express $f(x)$ in partial fractions.

[5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^0 f(x) \, dx = 1 + \frac{1}{2} \ln\left(\frac{3}{4}\right)$.

[5]

Question 3

An appropriate form for expressing $\frac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1} + \frac{B}{x-2},$$

where A and B are constants.

(a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\frac{4x}{(x+4)(x^2+3)},$ [1]

(ii) $\frac{2x+1}{(x-2)(x+2)^2}.$ [2]

(b) Show that $\int_3^4 \frac{3x}{(x+1)(x-2)} dx = \ln 5.$ [6]

Question 4

Show that $\int_0^7 \frac{2x+7}{(2x+1)(x+2)} dx = \ln 50$.

[7]