

Integration using Trig

Difficulty: Medium

Question Paper 2

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|------------|------------------------|
| Level | A Level |
| Subject | Maths Pure 3 |
| Exam Board | CIE |
| Topic | Integration |
| Sub-Topic | Integration using Trig |
| Difficulty | Medium |
| Booklet | Question Paper 2 |

Time allowed: 42 minutes

Score: /30

Percentage: /100

Grade Boundaries:

| A* | A | B | C | D | E |
|------|-----|-----|-----|-----|-----|
| >90% | 81% | 70% | 58% | 46% | 34% |

Question 1

(i) Prove the identity

$$\cot x - \cot 2x \equiv \operatorname{cosec} 2x. \quad [3]$$

(ii) Show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \cot x \, dx = \frac{1}{2} \ln 2.$ [3]

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \operatorname{cosec} 2x \, dx$, giving your answer in the form $a \ln b.$ [4]

Question 2

(i) Prove the identity $\cos 4\theta + 4 \cos 2\theta \equiv 8 \cos^4 \theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4 \cos 2\theta = 1$ for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$, [3]

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(b) find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4 \theta \, d\theta$. [3]

Question 3

- (i) Express $4 \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

- (a) solve the equation $4 \cos \theta + 3 \sin \theta = 2$ for $0 < \theta < 2\pi$, [4]

- (b) find $\int \frac{50}{(4 \cos \theta + 3 \sin \theta)^2} d\theta$. [3]