

Integration using Trig

Difficulty: Medium

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Integration
Sub-Topic	Integration using Trig
Difficulty	Medium
Booklet	Question Paper 1

Time allowed: 45 minutes

Score: /32

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Question 1

(i) Prove that if $y = \frac{1}{\cos \theta}$ then $\frac{dy}{d\theta} = \sec \theta \tan \theta$. [2]

(ii) Prove the identity $\frac{1 + \sin \theta}{1 - \sin \theta} = 2 \sec^2 \theta + 2 \sec \theta \tan \theta - 1$. [3]

(iii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \frac{1 + \sin \theta}{1 - \sin \theta} d\theta$. [4]

Question 2

- (i) Express $\cos \theta + \sqrt{3} \sin \theta$ in the form $R \cos (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2} \pi$, giving the exact values of R and α . [3]

- (ii) Hence show that $\int_0^{\frac{1}{2}\pi} \frac{1}{(\cos \theta + \sqrt{3} \sin \theta)^2} d\theta = \frac{1}{\sqrt{3}}$. [4]

Question 3

(a) Find $\int (4 + \tan^2 2x) dx$.

[3]

(b) Find the exact value of $\int_{\frac{1}{4}n}^{\frac{1}{2}n} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x} dx$.

[5]

Question 4

(i) By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$. [2]

(ii) Show that $\frac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

(iii) Deduce that $\frac{1}{(\sec x - \tan x)^2} \equiv 2 \sec^2 x - 1 + 2 \sec x \tan x$. [2]

(iv) Hence show that $\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2} dx = \frac{1}{4}(8\sqrt{2} - \pi)$. [3]