Calculus
Difficulty: Hard

Question Paper 1

Level	A Level
Subject	Maths Pure 3
Exam Board	CIE
Topic	Calculus
Difficulty	Hard
Booklet	Question Paper 1

Time allowed: **77 minutes**

Score: **/55**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E
>90%	81%	70%	58%	46%	34%

Question 1

Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \ln(1 + \sin 2x)$, [2]

(ii) $y = \frac{\tan x}{x}$. [2]

Question 2

For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \frac{1 + x^2}{1 + e^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

(a) Show that $\int_2^4 4x \ln x \, dx = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\int_0^{\frac{1}{24}\pi} \cos^3 4x \, dx$. [5]

Question 4

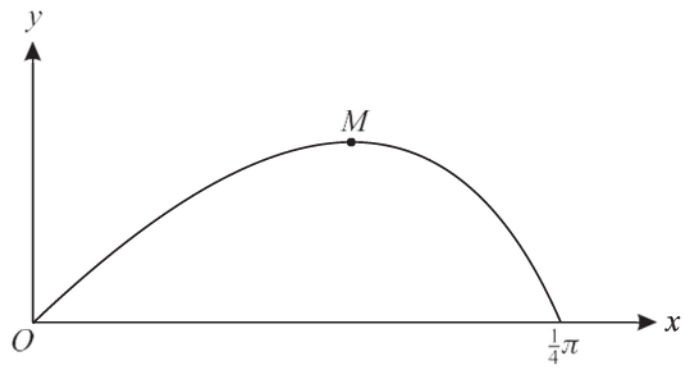
Let $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,dx$.

(i) Using the substitution $x = \cos^2\theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2\cos^2\theta\,d\theta$. [4]

(ii) Hence find the exact value of I. [4]

(i) Show that $\dfrac{2\sin x - \sin 2x}{1 - \cos 2x} = \dfrac{\sin x}{1 + \cos x}$. [4]

(ii) Hence, showing all necessary working, find $\displaystyle\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2\sin x - \sin 2x}{1 - \cos 2x}\,dx$, giving your answer in the form $\ln k$. [4]



The diagram shows the curve $y = x \cos 2x$ for $0 \le x \le \frac{1}{4}\pi$. The point M is a maximum point.

(i) Show that the x-coordinate of M satisfies the equation $1 = 2x \tan 2x$. [3]

(ii) The equation in part **(i)** can be rearranged in the form $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$. Use the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right),$$

with initial value $x_1 = 0.4$, to calculate the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(iii) Use integration by parts to find the exact area of the region enclosed between the curve and the x-axis from 0 to $\frac{1}{4}\pi$. [5]

Let $I = \int_0^1 \frac{x^2}{\sqrt{(4 - x^2)}}\, dx$.

(i) Using the substitution $x = 2 \sin \theta$, show that

$$I = \int_0^{\frac{1}{6}\pi} 4 \sin^2 \theta \, d\theta.$$ **[3]**

(ii) Hence find the exact value of I. [4]