Solving Differential Equations
Difficulty: Medium

Question Paper 2

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Integration
Sub-Topic	Solving Differential Equations
Difficulty	Medium
Booklet	Question Paper 2

Time allowed: **47 minutes**

Score: **/39**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t.

(4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$.

(3)

In an improved model the differential equation is given as

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \lambda P \cos \lambda t ,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t.

(4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.

(3)

(Total 14 marks)

(a) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions. **(3)**

(b) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x > 1$. **(3)**

(c) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$. **(6)**

(Total 12 marks)

(a) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{2}P(P-2)\cos 2t\,,\; t \geq 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$, **(3)**

(b) solve this differential equation to show that

$$P = \frac{6}{3 - \mathrm{e}^{\frac{1}{2}\sin 2t}}$$ **(7)**

(c) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures. **(3)**

(Total 13 marks)