Trapezium Rule
Difficulty: Medium

Question Paper 3

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Integration
Sub-Topic	Trapezium Rule
Difficulty	Medium
Booklet	Question Paper 3

Time allowed: **73 minutes**

Score: **/61**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

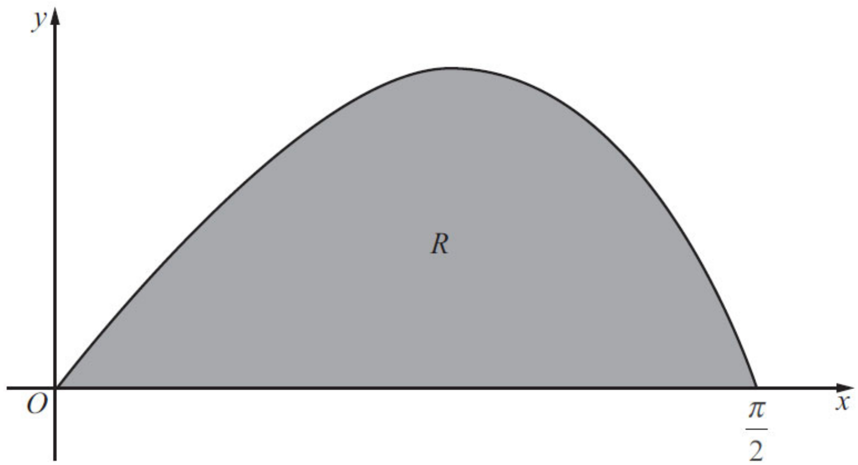

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \le x \le \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(a) Complete the table above giving the missing value of y to 5 decimal places. **(1)**

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places. **(3)**

(c) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2 \sin 2x}{(1 + \cos x)} \, dx = 4 \ln(1 + \cos x) - 4 \cos x + k$$

where k is a constant. **(5)**

(d) Hence calculate the error of the estimate in part (b), giving your answer to 2 significant figures. **(3)**

(Total 12 marks)

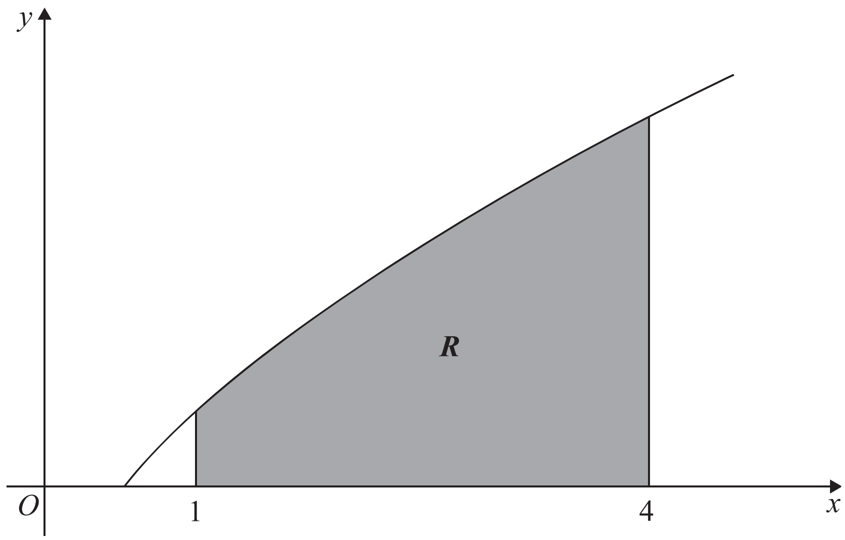

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places. **(4)**

(b) Find $\int x^{\frac{1}{2}} \ln 2x \, dx$. **(4)**

(c) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants. **(3)**

(Total 11 marks)

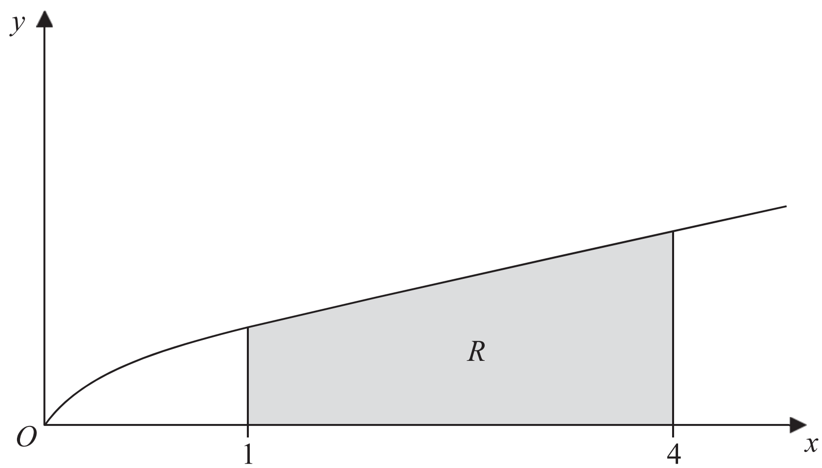

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places. **(1)**

x	1	2	3	4
y	0.5	0.8284		1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places. **(3)**

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

(Total 12 marks)

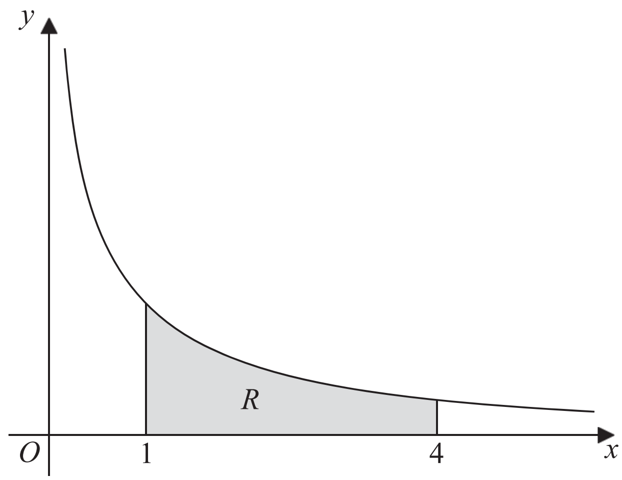

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$

The table below shows corresponding values of x and y for $y = \dfrac{10}{2x + 5\sqrt{x}}$

x	1	2	3	4
y	1.42857	0.90326		0.55556

(a) Complete the table above by giving the missing value of y to 5 decimal places. **(1)**

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places. **(3)**

(c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of R. **(1)**

(d) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,\mathrm{d}x$$ **(6)**

(Total 11 marks)

The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leq x \leq 4$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 \left(8 - 2^{x-1}\right) \mathrm{d}x$ **(3)**

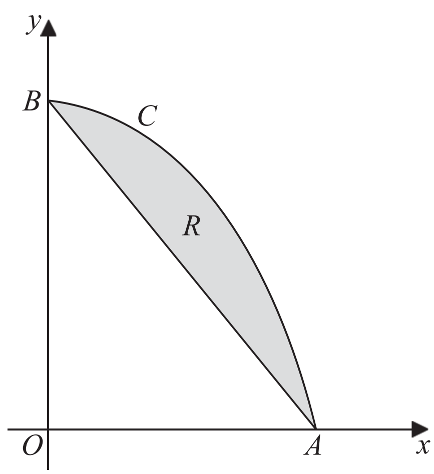

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}$, $0 \leq x \leq 4$

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R. **(2)**

(Total 6 marks)

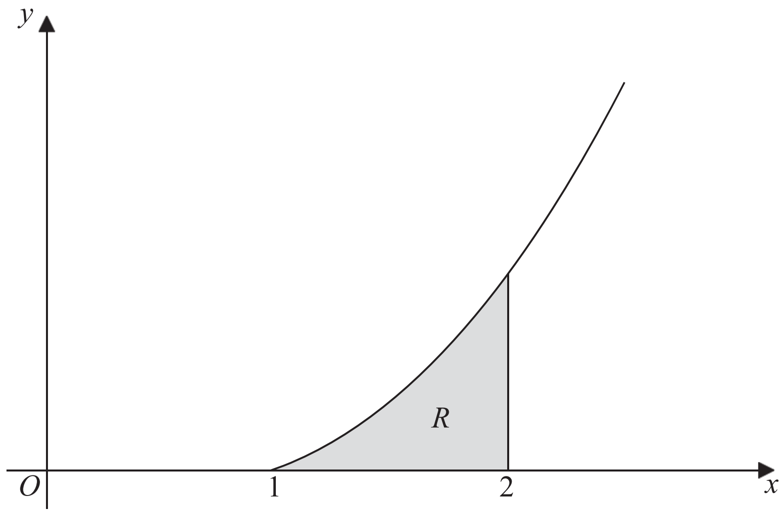

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x$, $x \geq 1$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$

The table below shows corresponding values of x and y for $y = x^2 \ln x$

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(a) Complete the table above, giving the missing value of y to 4 decimal places. **(1)**

(b) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places. **(3)**

(c) Use integration to find the exact value for the area of R. **(5)**

(Total 9 marks)