Iteration
Difficulty: Easy

Question Paper 2

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Numerical Methods
Sub-Topic	Iteration
Difficulty	Easy
Booklet	Question Paper 2

Time allowed: **56 minutes**

Score: **/47**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

$$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leqslant x < 2\pi$$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$

(2)

The equation $f(x) = 0$ can be written as $x = \left[\arcsin\left(1 - 0.5x\right)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin\left(1 - 0.5x_n\right)\right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

(3)

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

(3)

(Total 8 marks)

$$f(x) = \ln(x+2) - x + 1, \quad x > -2, \quad x \in \mathbb{R}.$$

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2\ 5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

(Total 7 marks)

$$f(x) = x^3 + 3x^2 + 4x - 12$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3-x)}{(3+x)}\right)}, \quad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, \; n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3. **(3)**

The root of $f(x) = 0$ is a.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(3)

(Total 9 marks)

$$g(x) = e^{x-1} + x - 6$$

(a) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2$$

is used to find an approximate value for α.

(b) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(c) By choosing a suitable interval, show that $a = 2.307$ correct to 3 decimal places.

(3)

(Total 8 marks)

$$\mathrm{f}(x) = x^4 - 8x^2 + 2$$

(a) Show that the equation f(x) = 0 can be written as $x = \sqrt{ax^4 + b}, x > 0$, where a and b are constants to be found.

(2 marks)

Let $x_0 = 1.5$.

(b) Use the iteration formula $x_{n+1} = \sqrt{ax_n^4 + b}$, together with your values of a and b from part (a), to find, to 4 decimal places, the values of x_1, x_2, x_3 and x_4.

(2 marks)

A root of f(x) = 0 is α. By choosing a suitable interval,

(c) prove that $\alpha = -2.782$ to 3 decimal places.

(3 marks)

(Total 7 marks)

Question 6

$$f(x) = x^3 - 2x - 5.$$

(a) Show that there is a root a of $f(x) = 0$ for x in the interval $[2, 3]$.

(2)

The root a is to be estimated using the iterative formula

$$x_{n+1} = \sqrt{\left(2 + \frac{5}{x_n}\right)}, \quad x_0 = 2.$$

(b) Calculate the values of x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures. **(3)**

(c) Prove that, to 5 significant figures, a is 2.0946. **(3)**

(Total 8 marks)