Algebra, Differentiation and Numerical Methods
Difficulty: Hard

Question Paper 2

Level	A Level only
Subject	Maths - Pure
Exam Board	Edexcel
Topic	Algebra, Differentiation and Numerical Methods
Sub-Topic	
Difficulty	Hard
Booklet	Question Paper 2

Time allowed: **66 minutes**

Score: **/55**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>76%	61%	52%	42%	33%	23%	<23%

$$f(x) = 1 - \frac{3}{x+2} + \frac{3}{(x+2)^2}, \; x \neq -2.$$

(a) Show that $f(x) = \dfrac{x^2 + x + 1}{(x+2)^2}, \; x \neq -2.$

(4)

(b) Show that $x^2 + x + 1 > 0$ for all values of x.

(3)

(c) Show that $f(x) > 0$ for all values of x, $x \neq -2$.

(1)

(Total 8 marks)

(a) Sketch the curve with equation $y = \ln x$.

(2)

(b) Show that the tangent to the curve with equation $y = \ln x$ at the point (e, 1) passes through the origin.

(3)

(c) Use your sketch to explain why the line $y = mx$ cuts the curve $y = \ln x$ between $x = 1$ and $x = \mathrm{e}$ if $0 < m < \frac{1}{\mathrm{e}}$.

(2)

Taking $x_0 = 1.86$ and using the iteration $x_{n+1} = e^{\frac{1}{3}x_n}$,

(d) calculate x_1, x_2, x_3, x_4 and x_5, giving your answer to x_5 to 3 decimal places.

(3)

The root of $\ln x - \frac{1}{3}x = 0$ is α.

(e) By considering the change of sign of $\ln x - \frac{1}{3}x$ over a suitable interval, show that your answer for x_5 is an accurate estimate of α, correct to 3 decimal places.

(3)

(Total 13 marks)

The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p.

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

(Total 7 marks)

The number of bacteria, N, present in a liquid culture at time t hours after the start of a scientific study is modelled by the equation

$$N = 5000(1.04)^t, \qquad t \geq 0$$

where N is a continuous function of t.

(a) Find the number of bacteria present at the start of the scientific study.

(1)

(b) Find the percentage increase in the number of bacteria present from $t = 0$ to $t = 2$

(2)

Given that $N = 15000$ when $t = T$,

(c) find the value of $\frac{\mathrm{d}N}{\mathrm{d}t}$ when $t = T$, giving your answer to 3 significant figures.

(4)

(Total 7 marks)

A scientist is studying a population of mice on an island.

The number of mice, N, in the population, t months after the start of the study, is modelled by the equation

$$N = \frac{900}{3 + 7e^{-0.25t}}, \quad t \in \mathbb{R}, \quad t \geqslant 0$$

(a) Find the number of mice in the population at the start of the study.

(1)

(b) Show that the rate of growth $\frac{dN}{dt}$ is given by $\frac{dN}{dt} = \frac{N(300 - N)}{1200}$

(4)

The rate of growth is a maximum after T months.

(c) Find, according to the model, the value of T.

(4)

According to the model, the maximum number of mice on the island is P.

(d) State the value of P.

(1)

(Total 10 marks)

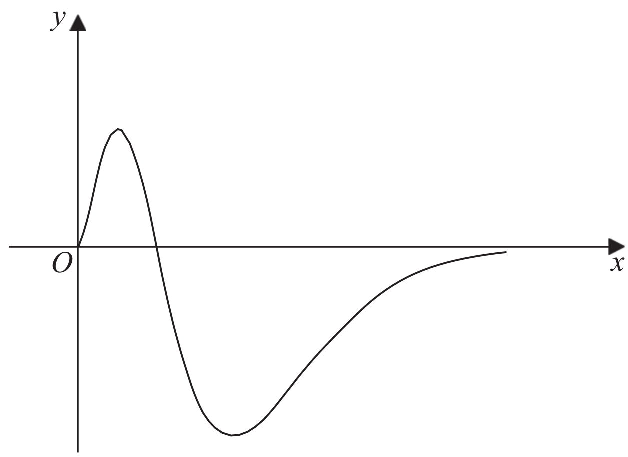

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geq 0$$

(a) Show that $g'(x) = f(x)\,e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g.

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

(Total 10 marks)