CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 *EITHER*: State or imply non-modular inequality $(4x + 3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$ M1

Obtain a critical value, e.g. –1 A1

Obtain a second critical value, e.g. $-\frac{3}{5}$ A1

State final answer $x < -1, x > -\frac{3}{5}$ A1

OR: Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection B1

Obtain the critical value $-\frac{3}{5}$ similarly B2

State final answer $x < -1, x > -\frac{3}{5}$ B1 [4]

[Do not condone $\leq$ or $\geq$.]

2 Use law for the logarithm of a product, quotient or power M1

Use ln e = 1 or exp(l) = 3 M1

Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = ex^3$ A1

Rearrange as $y = (ex^3 - 1)^{-1}$, or equivalent A1 [4]

3 Use correct tan 2*A* formula and cot x = 1/tan x to form an equation in tan x M1

Obtain a correct horizontal equation in any form A1

Solve an equation in $\tan^2 x$ for x M1

Obtain answer, e.g. 40.2° A1

Obtain second answer, e.g. 139.8°, and no other in the given interval A1√ [5]

[Ignore answers outside the given interval.]

[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]

[SR: For the answer $x = 90°$ give B1 and A1 for one of the other angles.]

4 **(i)** State $R = 2$ B1

Use trig formula to find α M1

Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen A1 [3]

(ii) Substitute denominator of integrand and state integral $k \tan(x - \alpha)$ M1*

State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$ A1√

Substitute limits M1 (dep*)

Obtain the given answer correctly A1 [4]

5 **(i)** Substitute $x = -\frac{1}{2}$, or divide by $(2x + 1)$, and obtain a correct equation, e.g. $a - 2b + 8 = 0$ B1

Substitute $x = \frac{1}{2}$ and equate to 1, or divide by $(2x - 1)$ and equate constant remainder to 1 M1

Obtain a correct equation, e.g. $a + 2b + 12 = 0$ A1

Solve for a or for b M1

Obtain $a = -10$ and $b = -1$ A1 [5]

(ii) Divide by $2x^2 - 1$ and reach a quotient of the form $4x + k$ M1

Obtain quotient $4x - 5$ A1

Obtain remainder $3x - 2$ A1 [3]

6 **(i)** State the correct derivatives $2e^{2x-3}$ and $2/x$ B1

Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$ M1

Obtain the given result correctly (or work *vice versa*) A1 [3]

(ii) Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent M1

Complete the argument with correct calculated values A1 [2]

(iii) Use the iterative formula correctly at least once M1

Obtain final answer 1.35 A1

Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355) A1 [3]

7 **(i)** Show that $a^2 + b^2 = (a + ib)(a - ib)$ B1

Show that $(a + ib - k\mathrm{i})^* = a - ib + k\mathrm{i}$ B1 [2]

(ii) Square both sides and express the given equation in terms of z and z^* M1

Obtain a correct equation in any form, e.g. $(z - 10\mathrm{i})(z^* + 10\mathrm{i}) = 4(z - 4\mathrm{i})(z^* + 4\mathrm{i})$ A1

Obtain the given equation A1

Either express $|z - 2i| = 4$ in terms of z and z^* or reduce the given equation to the form $|z - u| = r$ M1

Obtain the given answer correctly A1 [5]

(iii) State that the locus is a circle with centre 2i and radius 5 B1 [1]

8 **(i)** Separate variables correctly and integrate at least one side — M1
Obtain term ln t, or equivalent — B1
Obtain term of the form $a \ln(k - x^3)$ — M1
Obtain term $-\frac{2}{3}\ln(k - x^3)$, or equivalent — A1
EITHER: Evaluate a constant or use limits $t = 1$, $x = 1$ in a solution containing $a \ln t$ and $b \ln(k - x^3)$ — M1*
Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k - x^3) + \frac{2}{3}\ln(k - 1)$ — A1
Use limits $t = 4$, $x = 2$, and solve for k — M1(dep*)
Obtain $k = 9$ — A1

OR: Using limits $t = 1$, $x = 1$ and $t = 4$, $x = 2$ in a solution containing $a \ln t$ and $b \ln (k - x^3)$ obtain an equation in k — M1*
Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k - 8) + \frac{2}{3}\ln(k - 1)$ — A1
Solve for k — M1(dep*)
Obtain $k = 9$ — A1

Substitute $k = 9$ and obtain $x = (9 - 8t^{-\frac{3}{2}})^{\frac{1}{3}}$ — A1 [9]

(ii) State that x approaches $9^{\frac{1}{3}}$, or equivalent — B1√ [1]

9 **(i)** Use product rule — M1
Obtain correct derivative in any form, e.g. $4\sin 2x \cos 2x \cos x - \sin^2 2x \sin x$ — A1
Equate derivative to zero and use a double angle formula — M1*
Reduce equation to one in a single trig function — M1(dep*)
Obtain a correct equation in any form,
e.g. $10 \cos^3 x = 6 \cos x$, $4 = 6 \tan^2 x$ or $4 = 10 \sin^2 x$ — A1
Solve and obtain $x = 0.685$ — A1 [6]

(ii) Using $\mathrm{d}u = \pm \cos x \, \mathrm{d}x$, or equivalent, express integral in terms of u and $\mathrm{d}u$ — M1
Obtain $\int 4u^2(1 - u^2)\,\mathrm{d}u$, or equivalent — A1
Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$ — M1
Obtain answer $\frac{8}{15}$ (or 0.533) — A1 [4]

10 **(i)** Equate scalar product of direction vector of l and p to zero — M1
Solve for a and obtain $a = -6$ — A1 [2]

(ii) Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$
or $(1 - \mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$ — B1
Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ — M1
Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a - 4) = 0$
or $(1 + \mu)(a - 4) = 0$ — A1
Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied — A1 [4]

(iii) Using the correct process for the moduli, divide scalar product of direction vector if l and normal to p by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation M1*

Use $\frac{2}{\sqrt{5}}$ as sine of the angle A1

State equation in any form, e.g. $\dfrac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}} = \dfrac{2}{\sqrt{5}}$ A1

Solve for a M1 (dep*)

Obtain answers for $a = 0$ and $a = \dfrac{60}{31}$, or equivalent A1 [5]

[Allow use of the cosine of the angle to score M1M1.]